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Abstract

This paper studies the volatility implications of anticipated cost-push shocks (i.e. news shocks) in a New Keynesian model under optimal unrestricted monetary policy with forward-looking rational expectations (RE) and backward-looking boundedly rational expectations (BRE). If the degree of backward-looking price setting behavior is sufficiently small (large), anticipated cost-push shocks lead to a higher (lower) volatility in the output gap and in the central bank's loss than an unanticipated shock of the same size. The inversion of the volatility effects of news shocks between rational and boundedly rational expectations follows from the inverse relation between the price-setting behavior and the optimal monetary policy. By contrast, if the central bank does not optimize and follows a standard Taylor-type rule and the price setters are purely (forward-) backward-looking, the volatility of the economy is (increasing with) independent of the anticipation horizon. The volatility results for the inflation rate are ambiguous.

JEL classification: E32, E52

Keywords: Anticipated shocks, Optimal monetary policy, Bounded rationality, Volatility

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1 Introduction

Several empirical studies emphasize the importance of news shocks for business cycle fluctuations. These shocks materialize in the future, but their size and maturity time is anticipated in advance by the agents. Most prominently, Schmitt-Grohé and Uribe (2012) find in an estimated real business cycle model that about 50 percent of economic fluctuations can be attributed to anticipated disturbances.[1]

A theoretical branch of the literature indicates that news shocks destabilize the economy, i.e. lead to a higher volatility than unanticipated shocks of the same form. Fève et al. (2009) demonstrate in a purely forward-looking rational expectations model that news shocks increase the volatility with increasing length of anticipation. With both backward- and forward-looking expectations, the volatility results are ambiguous as it is shown by Winkler and Wohltmann (2012) in an univariate model. However, they find that the anticipation of cost shocks – as considered here – greatly amplifies the volatility of all key macroeconomic variables in the estimated model of Smets and Wouters (2003).[2]

These (empirical and theoretical) findings rely on the assumption of forward-looking rational expectations. By contrast, under purely backward-looking boundedly rational expectations, the volatility is independent of the anticipation horizon.[3] Bounded rationality assumes that agents have cognitive limitations and use simple heuristics (rule of thumbs) to guide their behavior and are recently under growing investigation.[4]

In light of these findings, our paper contributes to the existing literature in three ways:

[1] Their finding is supported by several VAR-based studies including Beaudry and Lucke (2010) and Barsky and Sims (2011). Beaudry and Portier (2006) and Jaimovich and Rebelo (2009) demonstrate that news shocks may help to explain recessions without relying on technological regress. However, there is no consensus about the importance of news shocks. Studies that find that news shocks only play a minor role include Fujiwara et al. (2011) and Forni et al. (2014). Kahn and Tsoukalas (2012) find in a structural DSGE model that news shocks account for less than 15 percent of the variance in output growth, but explain more than 60 percent in hours worked and inflation. For an extensive literature review on news shocks, readers are referred to Barsky and Portier (2013).

[2] Further related to this branch of literature is the paper by Offick and Wohltmann (2013), who study the properties of the lag polynomial associated with news shocks.

[3] To see this, consider the model $y_t = \rho y_{t-1} + \varepsilon_{t-q}$, where $\varepsilon_{t-q} \sim N(0, \sigma^2)$ is an i.i.d. news shock that is anticipated q periods in advance. Assuming stationarity, the variance of this model is given by $Var(y_t) = \sigma^2/(1-\rho^2)$, i.e. independent of q.

[4] De Grauwe (2012) e.g. combines boundedly rational expectations with the theory of discrete choice, which allows agents to choose between a set of heuristics. His model is able to create non-normally distributed movements in output growth. Lengnick and Wohltmann (2014) use a similar approach in a New Keynesian model with financial markets.

First, we combine the theory of news shocks and optimal monetary policy in a New Keynesian framework. Second, we study the (de)stabilizing effects of anticipated cost shocks in a multivariate environment. Third, we analyze how the relative volatility results of news shocks change if rational expectations are replaced by boundedly rational expectations. We introduce bounded rationality by assuming that a fraction of price setters have static expectations as in Leitemo (2008). We provide analytical results for the limit case of purely forward- and purely backward-looking price setting behavior.

So far, optimal monetary policy has been studied almost exclusively in the presence of unanticipated disturbances.[5] One exception is the study of Winkler and Wohltmann (2011), who analyze optimal simple interest rules. They find that the inclusion of forward-looking elements in an instrument rule is welfare enhancing in the case of anticipated shocks.[6] However, they focus on purely forward-looking private expectations and the resulting welfare effects. By contrast, we study the relation between news shocks, volatility, optimal unrestricted monetary policy, and (boundedly) rational expectations.

2 News shocks and optimal monetary policy

We assume that the inflation rate is governed by a standard hybrid New Keynesian Phillips curve of the form

$$\pi_t = \beta(1-\phi_\pi)E_t\pi_{t+1} + \beta\phi_\pi\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \tag{1}$$

where π_t and x_t are the inflation rate and the output gap measured as percentage deviations from the steady state, respectively. ϕ_π measures the degree to which price setters are boundedly rational and have backward-looking expectations. For $\phi_\pi = 0$ ($\phi_\pi = 1$), the price-setting behavior is purely forward-looking (backward-looking). ε_{t-q} is a white noise cost-push shock

[5] This includes Leitemo (2008), who finds an inverse relation between the private pricing behavior and the optimal monetary strategy. If the private sector is backward-looking, monetary policy should be forward-looking, and vice versa. This general result also holds for news shocks.

[6] Further noteworthy is the paper by Winkler and Wohltmann (2009), who show how to solve rational expectations models with news shock under optimal monetary policy.

with unit variance which is anticipated q periods in advance.[7] The shock is unanticipated for $q = 0$.

For convenience, we assume that the central bank aims to minimize the weighted sum of variance of the inflation rate and the output gap. The central bank's loss is given by

$$Loss_q = \lambda_1 Var_q(\pi_t) + \lambda_2 Var_q(x_t) \tag{2}$$

As in Leitemo (2008), the optimal targeting rule then includes forward- and backward-looking elements and reads as

$$\pi_t = -\frac{\lambda_2}{\lambda_1 \kappa}(x_t - x_{t-1}) - \frac{\lambda_2}{\lambda_1 \kappa}\phi_\pi x_{t-1} + \frac{\lambda_2}{\lambda_1 \kappa}\beta^2 \phi_\pi E_t x_{t+1} \tag{3}$$

The central bank optimization is independent of the form of the IS equation and of the lead time q. Equations (1) and (3) fully describe the dynamics of the output gap and the inflation rate.

Before we turn to the general case of hybrid private price-setting behavior, we discuss the limit case of purely forward-looking price setting. Note that in both limit cases ($\phi_\pi = 0$ and $\phi_\pi = 1$) the system remains hybrid. This is due to the inverse relation between the price-setting behavior and the optimal monetary strategy as described in Leitemo (2008).

2.1 Purely forward-looking price setters

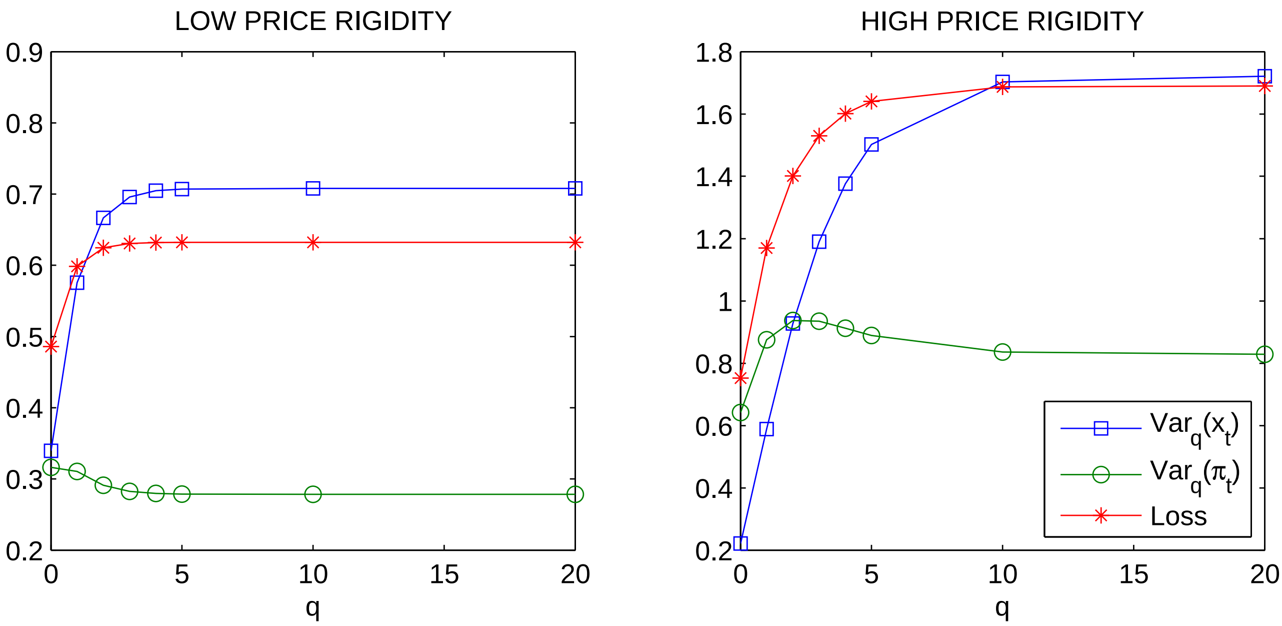
For $\phi = 0$, the system can be reduced to an univariate hybrid equation of the form

$$x_t = aE_t x_{t+1} + bx_{t-1} + c\varepsilon_{t-q} \tag{4}$$

with $a = \beta b$, $b = \lambda_2/(\lambda_2(1+\beta) + \lambda_1\kappa^2)$, and $c = -\lambda_1\kappa/(\lambda_2(1+\beta) + \lambda_1\kappa^2)$. Since $1 > \beta > 0$, $sgn(a) = sgn(b)$. This implies that the variance of x_t is unambiguously increasing in q as it is shown by Winkler and Wohltmann (2012).

[7] Note that we limit our discussion to cost-push shocks for which the central bank faces a trade off between output and inflation stabilization even without instrument target as considered here. This type of shock is also found to be highly relevant for business cycle fluctuations, see e.g. Schmitt-Grohé and Uribe (2012).

Figure 1: Loss and variances in the case of purely forward-looking price setting



Note: Parameters are set to $\beta = 0.99$, $\sigma = \eta = 2$, $\kappa = (\sigma + \eta)(1 - \omega)(1 - \omega\beta)/\omega$, $\lambda_1 = 1$, $\lambda_2 = 0.5$. Under low (high) price rigidity, the Calvo parameter ω is set to 0.7 (0.8), implying $\kappa = 0.2$ ($\kappa = 0.53$). Assuming continuity, the maximum in the inflation variance is reached in $q^* = 0.08$ ($q^* = 2.0$).

The volatility of the inflation rate, on the other hand, may also be decreasing in q. Its variance is given by

$$Var(\pi_t) = \frac{2\beta_0^2}{(1+\alpha)(1+\delta)(1-\alpha\delta)} \left(\frac{\lambda_2}{\lambda_1 \kappa}\right)^2 \left[1 - \frac{1-\alpha\delta}{\alpha - \delta}\delta^{2(q+1)} + \frac{(1-\alpha)(1+\delta)\delta\alpha}{\alpha-\delta}(\alpha\delta)^q\right] \quad (5)$$

where $|\alpha| < 1$ is the stable root of $\alpha_{1,2} = \left(1 \pm \sqrt{1-4ab}\right)/(2a)$, $\beta_0 = c/(1-a\alpha)$, and $\delta = a/(1-a\alpha)$.[8] An unanticipated shock may generate a higher inflation volatility than a cost-push shock that is anticipated in the infinite past:

$$Var_{q=0}(\pi_t) > Var_{q\to\infty}(\pi_t) \quad \text{if} \quad \frac{\lambda_1\kappa^2}{\lambda_2} > \sqrt{1+4\beta} - (1+\beta) \qquad (\lambda_2 > 0) \quad (6)$$

The reason for the ambiguity in the inflation volatility are two opposing effects: On the one hand, the longer the length of anticipation, the higher is the variance of the output gap, which – in isolation – also leads to a higher variance in inflation. On the other hand, the response of

[8]Note that the output gap can be written as an ARMA(1,q) process of the form $x_t = \alpha x_{t-1} + \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t+k-q}$. A stable solution requires $|\alpha| < 1$. For a full derivation of the results under purely forward-looking price setting, see Appendix A and B.

the output gap becomes smoother, i.e. x_t is more autocorrelated, with increasing q. Since the inflation rate depends via the targeting rule on the change in the output gap, this reduces – in isolation – the variance of inflation.[9] Condition (6) does not imply that an anticipated shock gives a lower inflation volatility for all anticipation horizons. That is, the inflation variance may not be monotonic in q. The maximum is reached in $q = \max(q^*, 0)$ where[10]

$$q^* = \frac{1}{\log\alpha - \log\delta}\left\{\log\frac{2\delta(1-\alpha\delta)}{(1-\alpha)\alpha(1+\delta)} + \log\frac{\log\delta}{\log\alpha\delta}\right\} \tag{7}$$

Despite the fact that the variance of inflation may be decreasing in q, the loss (2) is always increasing in q. Only under strict inflation targeting ($\lambda_2 = 0$) does the central bank perfectly stabilize the inflation rate and the loss is zero, independently from q.

Figure 1 illustrates the above results for high and low price rigidity. Under low (high) price rigidity, the Phillips curve parameter κ is relatively large (small) such that condition (6) is (not) satisfied.

2.2 Hybrid price-setting behavior

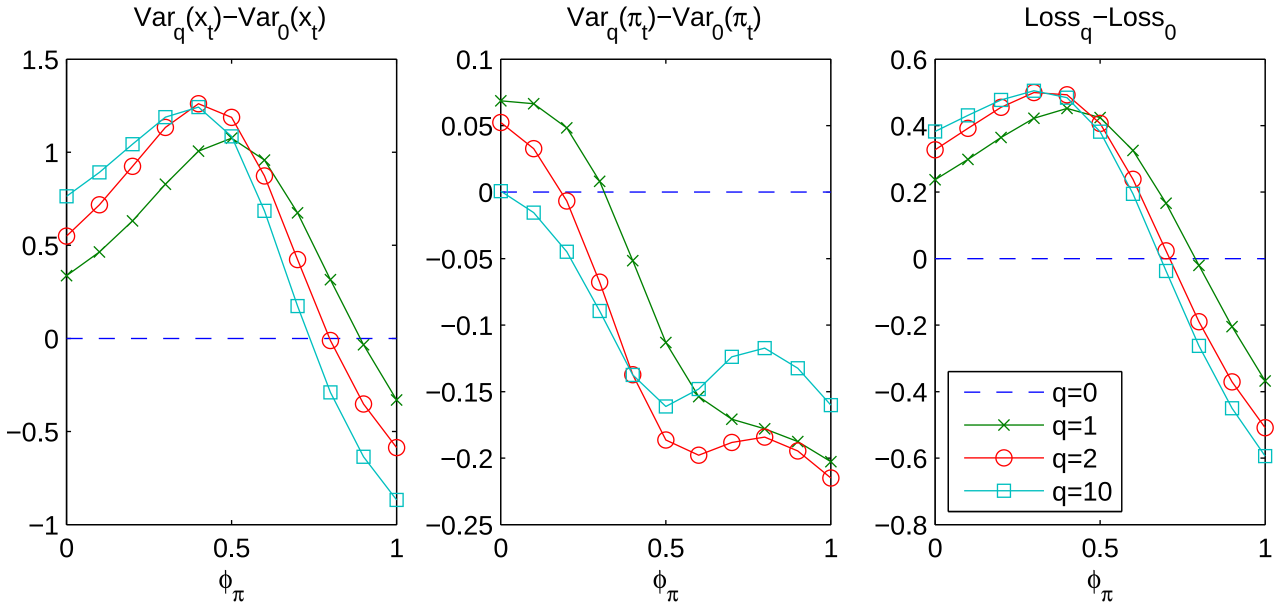
If we allow for backward-looking price-setting behavior (i.e. $\phi_\pi > 0$), the results under purely forward-looking price setting of the previous subsection may be reversed. This reversion can be seen in figure 2 and 3. Figure 2 shows the differences in the loss and in the volatilities of the output gap and the inflation rate between an anticipated and an unanticipated cost shock for different degrees of hybridity and anticipation horizons. If ϕ_π – the degree of backward-lookingness – is sufficiently large, all three differences are negative for arbitrary anticipation horizons. Contrarily to the output gap, the volatility in inflation may not be monotonic in q for $\phi_\pi > 0$.

Figure 3 compares the volatilities and the loss of an unanticipated ($q = 0$) and an anticipated ($q = 20$) shock – additionally to ϕ_π – for different degrees of price rigidity ω and for different weights λ_2 the central bank puts on output stabilization. We find that the volatility in the output gap and the loss are less variant to changes in λ_2 and ω. In case of purely backward-

[9] The two opposing effects can be directly seen by taking the variance of the targeting rule: $Var(\pi_t) = 2\lambda_2^2/(\lambda_1\kappa)^2[Var(x_t) - E(x_t x_{t-1})]$, where both $Var(x_t)$ and $E(x_t x_{t-1})$ are increasing in q.

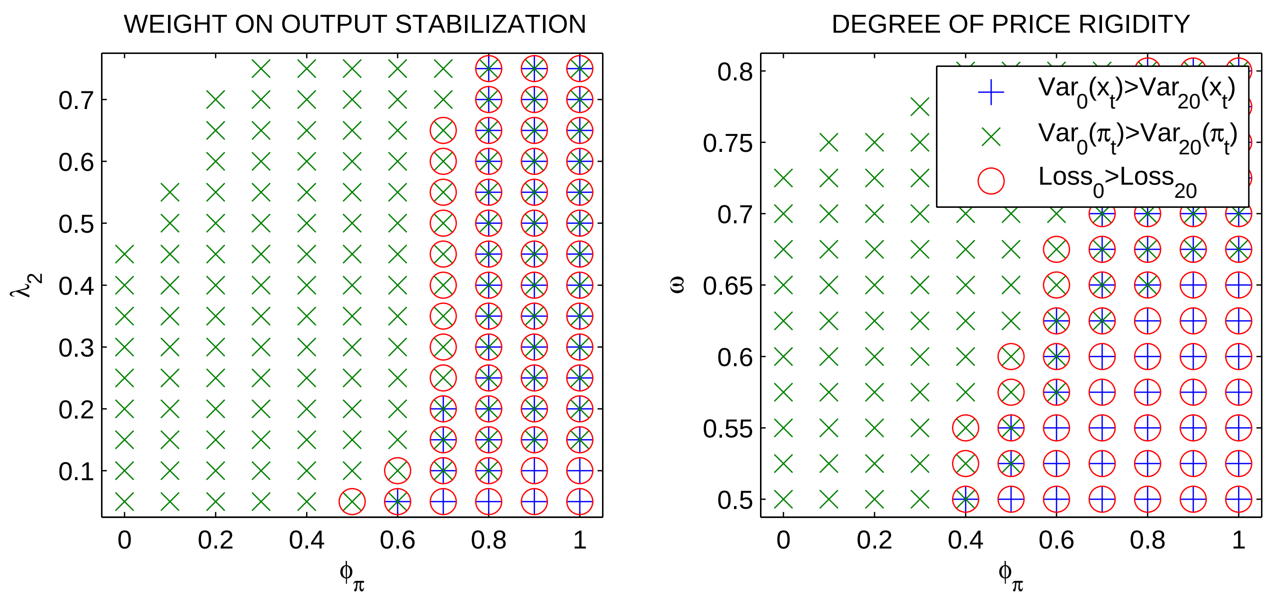
[10] Note that equation (7) assumes that q is continuous.

Figure 2: Loss and variances for different degrees of hybridity



Parameter calibration: $\beta = 0.99$, $\sigma = \eta = 2$, $\omega = 0.75$, $\kappa = (\sigma + \eta)(1 - \omega)(1 - \omega\beta)/\omega$, $\lambda_1 = 1$, $\lambda_2 = 0.5$.

Figure 3: Parameter sensitivity



Note: In left plot we set $\omega = 0.75$ and change the central bank's weight of output stabilization λ_2 ranging from 0.05 to 0.75. In the right plot we set $\lambda_2 = 0.5$ and change the degree of price rigidity ω ranging from 0.5 to 0.8. The remaining parameters are calibrated as follows: $\beta = 0.99$, $\sigma = \eta = 2$, $\kappa = (\sigma + \eta)(1 - \omega)(1 - \omega\beta)/\omega$, $\lambda_1 = 1$. Note that $\oplus$ means that both $Var_{20}(x_t)$ and $Loss_{20}$ are smaller, $\otimes$ means that both $Var_{20}(\pi_t)$ and $Loss_{20}$ are smaller , and the combination of all three symbols means that both variances and the loss are smaller for $q = 20$ than for $q = 0$.

looking price setting, the volatility in output and the loss is decreasing in q for all parameter constellations under consideration. Contrarily, the volatility results for the inflation rate are ambiguous for both limit cases.[11]

In summary, if ϕ_π is sufficiently large, it holds that: (i) The variance of the output gap and the loss decrease monotonically with increasing lead time q. (ii) The variance of the inflation rate is decreasing (increasing) in q if the weight λ_2 and/or the degree of price rigidity ω are sufficiently large (small). The reason for this inversion of volatility results is the inverse relation between the private pricing behavior and the optimal monetary policy strategy as described in Leitemo (2008).

3 Concluding remarks

This paper studies the volatility implications of anticipated cost-push shocks in a hybrid New Keynesian model with forward- and backward-looking price setters and optimal (unrestricted) monetary policy response. In particular, it is analyzed how the relative volatility results of news shocks under optimal monetary policy change if rational expectations are replaced by boundedly rational expectations.

We find that the destabilizing effects of anticipated cost-push shocks crucially depend on the type of private expectations. Under purely forward-looking rational expectations, the volatility in the output gap and the central bank's loss are unambiguously increasing with increasing anticipation horizon. Contrarily, under bounded rationality, we obtain the reversed result: If the degree of backward-looking price setting behavior is sufficiently large, the anticipation of cost-push shocks leads to a stabilization of the output gap and the central bank's loss. If – in addition – the central bank's weight on output stabilization and/or the degree of price rigidity is sufficiently large, we also obtain a stabilization of the inflation rate.

The inversion of the volatility effects of news shocks between rational and boundedly rational expectations follows from the optimization of the central bank. This optimization leads to an inverse relation between the price-setting behavior and the optimal monetary policy. By contrast, if the central bank follows an ad hoc or optimized standard Taylor-type rule and the

[11] Analytical results for this limit case can be found in the Appendix C.

price setters are purely (forward-) backward-looking, the volatility of the economy is (increasing with) independent of the anticipation horizon.[12]

Two remarks on the robustness of our results in order: First, without instrument target in the loss function of the central bank, the form of the targeting rule is independent of the form of the dynamic IS equation. Hence, our volatility results also hold for non-separable utility functions as in Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012). Second, we argue that our results also hold for more complex backward-looking price-setting behavior as in De Grauwe (2012). For reasons of space, we model boundedly rational expectations only as static expectations.[13]

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[12]Specifically, the Taylor-type rule must not contain any (backward-) forward-looking element for this to hold.

[13]We also studied the (de)stabilization effects of news shocks in a boundedly rational model with switching, in which the price setters are able to choose from a set of backward-looking expectations heuristics. The model setup is taken from Lengnick and Wohltmann (2014). Contrarily to the model in De Grauwe (2012) and Lengnick and Wohltmann (2014), we include forward-looking rational expectations through the optimal monetary strategy. We do not obtain qualitative changes in comparison to the model without purely backward-looking model without switching. Results are available upon request.

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Appendix

A Hybrid univariate model

A hybrid univariate model of the form

$$y_t = aE_t y_{t+1} + b y_{t-1} + c\varepsilon_{t-q} \tag{A.1}$$

with $\varepsilon_t \overset{i.i.d.}{\sim} N(0,\sigma^2)$ can be written as MA(∞) of the form

$$y_t = \sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t-s+k-q} = \sum_{s=0}^{\infty} \alpha^s h_{t-s} \quad \text{with} \quad h_t = \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t+k-q} \tag{A.2}$$

where $\alpha = \left(1 - \sqrt{1-4ab}\right)/(2a)$, $\beta_0 = c/(1-a\alpha)$, and $\delta = a/(1-a\alpha)$. The variance of y_t can be derived as follows:

$$Var(y_t) = \sum_{s=0}^{\infty}\sum_{\tilde{s}=0}^{\infty} \alpha^s \alpha^{\tilde{s}} \sum_{k=0}^{q}\sum_{\tilde{k}=0}^{q} \beta_0^2 \delta^k \delta^{\tilde{k}} E(\varepsilon_{t-s+k-q}\varepsilon_{t-\tilde{s}+\tilde{k}-q}) \tag{A.3}$$

$$= \beta_0^2 \sum_{s=0}^{\infty} \alpha^{2s} \sum_{k=0}^{q} \delta^{2k}\sigma^2 + 2\beta_0^2 \sum_{s=0}^{\infty}\sum_{j=0}^{q-1}\sum_{k=0}^{q-1-j} \alpha^{2s+j+1}\delta^{2k+j+1}\sigma^2 \tag{A.4}$$

$$= \beta_0^2 v_t \sigma^2 + 2\beta_0^2 w_t \sigma^2 \tag{A.5}$$

v_t and w_t can be simplified to

$$v_t = \sum_{s=0}^{\infty} \alpha^{2s} \sum_{k=0}^{q} \delta^{2k} = \frac{1}{1-\alpha^2}\frac{1-\delta^{2(q+1)}}{1-\delta^2} \tag{A.6}$$

$$w_t = \sum_{s=0}^{\infty}\sum_{j=0}^{q-1}\sum_{k=0}^{q-1-j} \alpha^{2s+j+1}\delta^{2k+j+1} \tag{A.7}$$

$$= \frac{\alpha\delta}{1-\delta^2}\sum_{s=0}^{\infty}\alpha^{2s}\sum_{j=0}^{q-1}(\alpha\delta)^j - \frac{\delta^{2(q+1)}}{1-\delta^2}\sum_{s=0}^{\infty}\alpha^{2s}\sum_{j=0}^{q-1}\left(\frac{\alpha}{\delta}\right)^{j+1} \tag{A.8}$$

$$= \frac{\alpha\delta}{1-\delta^2}\frac{1}{1-\alpha^2}\frac{1-(\alpha\delta)^q}{1-\alpha\delta} - \frac{\alpha\delta^{2(q+1)}}{1-\delta^2}\frac{1}{1-\alpha^2}\frac{1-\left(\frac{\alpha}{\delta}\right)^q}{\delta-\alpha} \tag{A.9}$$

In summary, the variance of y_t is given by $Var(y_t) = V(q)$ where

$$V(q) = \frac{\beta_0^2}{(1-\alpha^2)(1-\delta^2)} \left\{ 1 - \delta^{2(q+1)} + 2\frac{\alpha\delta}{1-\alpha\delta}\left[1-(\alpha\delta)^q\right] + 2\frac{\alpha}{\alpha-\delta}\left[\delta^{2(q+1)} - \delta^2(\alpha\delta)^q\right] \right\} \sigma^2 \tag{A.10}$$

Note that $V(q)$ can also be written as

$$V(q) = \frac{1}{1-\alpha^2}[Var(h_t) + 2\alpha Cov(x_{t-1}, h_t)] \tag{A.11}$$

where

$$Var(h_t) = \frac{\beta_0^2}{1-\delta^2}(1-\delta^{2(q+1)})\sigma^2 = \beta_0^2\sigma^2 \sum_{k=0}^{q} \delta^{2k} \tag{A.12}$$

$$Cov(x_{t-1}, h_t) = \frac{\beta_0^2}{1-\delta^2} \left\{ \frac{\delta}{1-\alpha\delta}\left[1-(\alpha\delta)^q\right] + \frac{1}{\alpha-\delta}\left[\delta^{2(q+1)} - \delta^2(\alpha\delta)^q\right] \right\} \sigma^2 \tag{A.13}$$

$$= \beta_0^2\sigma^2\delta \sum_{j=0}^{q-1} (\alpha\delta)^j \sum_{k=0}^{q-1-j} \delta^{2k} \tag{A.14}$$

B Purely forward-looking price setting

The model (1) and (3) in case of purely forward-looking price setting reads

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_{t-q} \tag{B.1}$$

$$\pi_t = -\frac{\lambda_2}{\lambda_1 \kappa}(x_t - x_{t-1}) \tag{B.2}$$

The output gap x_t can be written as hybrid univariate model equation of the form (A.1) with

$$a = \beta b \tag{B.3}$$

$$b = \frac{\lambda_2}{\lambda_2(1+\beta) + \lambda_1\kappa^2} \tag{B.4}$$

$$c = -\frac{\lambda_1\kappa_2}{\lambda_2(1+\beta) + \lambda_1\kappa^2} \tag{B.5}$$

Hence, the variance of x_t is given by $Var(x_t) = V(q)$, where

$$\delta = \frac{2a}{1+\sqrt{1-4ab}} \tag{B.6}$$

$$\alpha\delta = \frac{1-\sqrt{1-4ab}}{1+\sqrt{1-4ab}} \tag{B.7}$$

$$1-4ab = \frac{(1-\beta)^2 + 2(1+\beta)z + z^2}{(1+\beta+z)^2} \tag{B.8}$$

$$z = \frac{\lambda_1 \kappa^2}{\lambda_2} \tag{B.9}$$

Since $dVar_q(h_t)/dq > 0$ and $dCov_q(x_{t-1}, h_t)/dq > 0$, it holds $dVar_q(x_t)/dq > 0$.

The variance of the inflation rate can be deduced from the targeting rule (B.2):

$$Var(\pi_t) = 2\left(\frac{\lambda_2}{\lambda_1\kappa}\right)^2 (Var(x_t) - E(x_t x_{t-1})) \tag{B.10}$$

$$= \frac{2}{1+\alpha}\left(\frac{\lambda_2}{\lambda_1\kappa}\right)^2 [Var_q(h_t) - (1-\alpha)Cov_q(x_{t-1}, h_t)] \tag{B.11}$$

$$= \frac{2}{1+\alpha}\left(\frac{\lambda_2}{\lambda_1\kappa}\right)^2 \beta_0^2\sigma^2 \left[\sum_{k=0}^{q} \delta^{2k} - (1-\alpha)\delta \sum_{j=0}^{q-1} (\alpha\delta)^j \sum_{k=0}^{q-1-j} \delta^{2k}\right] \tag{B.12}$$

To derive the condition for $Var_{q=0}(\pi_t) > Var_{q\to\infty}(\pi_t)$, note that

$$Var_{q=0}(h_t) = \beta_0^2\sigma^2 \tag{B.13}$$

$$Cov_{q=0}(x_{t-1}, h_t) = 0 \tag{B.14}$$

$$Var_{q\to\infty}(h_t) = \frac{\beta_0^2}{1-\delta^2}\sigma^2 \tag{B.15}$$

$$Cov_{q\to\infty}(x_{t-1}, h_t) = \frac{\beta_0^2}{1-\delta^2}\frac{\delta}{1-\alpha\delta}\sigma^2 \tag{B.16}$$

Using the definitions (B.3) to (B.5), $Var_{q=0}(\pi_t) \overset{!}{>} Var_{q\to\infty}(\pi_t)$ is equivalent to

$$(1-\alpha)Cov_{q\to\infty}(x_{t-1}, h_t) > Var_{q\to\infty}h_t - Var_{q=0}h_t \quad \Leftrightarrow \quad \frac{1-\alpha}{1-\alpha\delta} > \delta \quad \Leftrightarrow \tag{B.17}$$

$$1 - \beta^{-1}a - 2\beta^{-1}a^2 > [(2+\beta^{-1})a - 1]\sqrt{1-4\beta^{-1}a^2} \quad \Leftrightarrow \tag{B.18}$$

$$(\beta^2-\beta) + (1+2\beta)\frac{\lambda_1\delta^2}{\lambda_2} + \frac{\lambda_1^2\kappa^4}{\lambda_2^2} > \left(\beta - \frac{\lambda_1\kappa^2}{\lambda_2}\right)\sqrt{(1-\beta)^2 + 2(1+\beta)\frac{\lambda_1\kappa^2}{\lambda_2} + \frac{\lambda_1^2\kappa^4}{\lambda_2^2}} \tag{B.19}$$

Let $z = \lambda_1\kappa^2/\lambda_2$, then inequality B.19 can be simplified to

$$z^2 + 2(1+\beta)z + \beta(\beta-2) > 0 \tag{B.20}$$

and holds if

$$z = \frac{\lambda_1\kappa^2}{\lambda_2} > \sqrt{1+4\beta} - (1+\beta) \tag{B.21}$$

Although the variance of the inflation rate may decrease with increasing anticipation horizon q, it can be shown that the loss

$$Loss_q = \lambda_1 Var_q(\pi_t) + \lambda_2 Var_q(x_t) \tag{B.22}$$

is always smaller for $q = 0$ than for $q \to \infty$. It holds:

$$Loss_{q\to\infty} = \left\{ \lambda_1 \frac{2}{1+\alpha} \left(\frac{\lambda_2}{\lambda_1\kappa}\right)^2 \frac{1-\delta}{1-\alpha\delta} \frac{1}{1-\delta^2} + \lambda_2 \frac{1}{1-\alpha^2} \frac{1}{1-\delta^2} \frac{1+\alpha\delta}{1-\alpha\delta} \right\} \beta_0^2\sigma^2 \tag{B.23}$$

$$Loss_{q=0} = \left\{ \lambda_1 \frac{2}{1+\alpha} \left(\frac{\lambda_2}{\lambda_1\kappa}\right)^2 + \lambda_2 \frac{1}{1-\alpha^2} \right\} \beta_0^2\sigma^2 \tag{B.24}$$

Then $J_{q\to\infty} \stackrel{!}{>} J_{q=0}$ is equivalent to

$$\frac{2\lambda_2}{\lambda_1\kappa}[1-\alpha(1+\delta)] < \frac{2\alpha+\delta(1-\alpha\delta)}{(1-\alpha)(1-\delta)} \quad \Leftrightarrow \quad 2b[\sqrt{1-4ab} - b] < b + b(1+\beta)\sqrt{1-4ab} \tag{B.25}$$

Since $b = 1/[1+\beta+z]$, $1-4ab = 1-4\beta/[1+\beta+z]^2$, (B.25) is equivalent to

$$0 < 4\beta(1-\beta)^2 + \beta(2-\beta)(1+\beta+z) + 4(1+\beta+z) \tag{B.26}$$

This inequality is always satisfied since $z = \lambda_1\kappa^2/\lambda_2 > 0$.

C Purely backward-looking price setting

The model (1) and (3) in case of purely backward-looking price setting reads

$$\pi_t = \beta\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \tag{C.1}$$

$$\pi_t = -\frac{\lambda_2}{\lambda_1\kappa}(x_t - \beta^2 E_t x_{t+1}) \tag{C.2}$$

The inflation rate can be written as a hybrid univariate equation of the form

$$\pi_t = aE_t\pi_{t+1} + b\pi_{t-1} + c(\varepsilon_{t-q} - \beta^2 E_t\varepsilon_{t-q+1}) \tag{C.3}$$

with $c = \varphi/(1+\varphi+\varphi\beta^3)$, $b = \beta c$, $a = \beta^2 c$, and $\varphi = \lambda_2/(\lambda_1\kappa^2)$. The system can again be written as

$$\pi_t = \sum_{s=0}^{\infty}\alpha^s h_{t-s} \tag{C.4}$$

where $\alpha = (1-\sqrt{1-4ab})/2a$ and

$$h_t = \sum_{k=0}^{q}\delta^k\beta_0\varepsilon_{t+k-q} - \beta^2\sum_{k=0}^{q-1}\delta^k\beta_0\varepsilon_{t+k-q+1} \tag{C.5}$$

The variance of the inflation rate can be derived as follows:

$$\begin{aligned} Var(\pi_t) &= E\left(\sum_{s=0}^{\infty}\alpha^s\sum_{k=0}^{q}\delta^k\beta_0\varepsilon_{t-s+k-q} - \beta^2\sum_{s=0}^{\infty}\alpha^s\sum_{k=0}^{q-1}\delta^k\beta_0\varepsilon_{t-s+k-q+1}\right)^2 &\text{(C.6)}\\ &= V(q) - 2\beta^2 Z + \beta^4 V(q-1) &\text{(C.7)}\end{aligned}$$

where $V(\cdot)$ is given by (A.10) and

$$Z = E\left[\left(\sum_{s=0}^{\infty}\alpha^s\sum_{k=0}^{q}\delta^k\beta_0\varepsilon_{t-s+k-q}\right)\left(\sum_{s=0}^{\infty}\alpha^s\sum_{k=0}^{q-1}\delta^k\beta_0\varepsilon_{t-s+k-q+1}\right)\right] \tag{C.8}$$

$$= E\left[\left(\sum_{s=0}^{\infty}\alpha^s\beta_0\varepsilon_{t-s-q}\right)\left(\sum_{s=0}^{\infty}\alpha^s\sum_{k=0}^{q-1}\delta^k\beta_0\varepsilon_{t-s+k-q+1}\right)\right] + \delta V(q-1) \tag{C.9}$$

$$= \frac{\alpha}{1-\alpha^2}\frac{1-(\alpha\delta)^q}{1-\alpha\delta}\beta_0^2\sigma^2 + \delta V(q-1) = \frac{\alpha}{1-\alpha^2}\beta_0^2\sigma^2\sum_{j=0}^{q-1}(\alpha\delta)^j + \delta V(q-1) \tag{C.10}$$

Then $Var_q(\pi_t)$ can be written as

$$Var(\pi_t) = V(q) - 2\beta^2\frac{\alpha}{1-\alpha^2}\frac{1-(\alpha\delta)^q}{1-\alpha\delta}\beta_0^2\sigma^2 + (\beta^4 - 2\beta^2\delta)V(q-1) \tag{C.11}$$

D Hybrid price-setting behavior

The model (1) and (3) in case of both forward- and backward-looking price setting can be written in matrix form

$$\Phi s_{t+1} = \Psi s_t + g\varepsilon_{t+1} \tag{D.1}$$

where $s_{t+1} = (\tilde{\eta}_{t+1}^{(q)}, \tilde{x}_{t+1}, \tilde{\pi}_{t+1}, E_t x_{t+1}, E_t\pi_{t+1})'$, $\tilde{\eta}_{t+1}^{(q)} = (\eta_{t+1}^{(0)}, \eta_{t+1}^{(1)}, \ldots, \eta_{t+1}^{(q-1)}, \eta_{t+1}^{(q)})'$ with $\eta_t^{(j)} = \varepsilon_{t-j}\,\forall\, j = 0, \ldots, q$ and $g = (1, 0, \ldots, 0)'$, and

$$\Phi = \begin{pmatrix} I_{q+3} & 0_{2\times(q+3)} \\ 0_{(q+3)\times 2} & \Phi_{22} \end{pmatrix} \tag{D.2}$$

$$\Psi = \begin{pmatrix} \Psi_{11} & 0_{(q+1)\times 5} \\ 0_{4\times q} & \Psi_{22} \end{pmatrix} \tag{D.3}$$

with $0_{n\times m}$ as $(n \times m)$-dimensional zero matrix, I_n as n-dimensional identity matrix, and

$$\Phi_{22} = \begin{pmatrix} \frac{\lambda_2\beta^2\phi_\pi}{\lambda_1\kappa} & 0 \\ 0 & \beta(1-\phi_\pi) \end{pmatrix} \tag{D.4}$$

$$\Psi_{11} = \begin{pmatrix} 0_{1\times q} \\ I_q \end{pmatrix} \tag{D.5}$$

$$\Psi_{22} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -(1-\phi_\pi)\frac{\lambda_2}{\lambda_1\kappa} & 0 & \frac{\lambda_2}{\lambda_1\kappa} & 1 \\ -1 & 0 & -\beta\phi_\pi & -\kappa & 1 \end{pmatrix} \tag{D.6}$$

Let $w_{t+1} = (\tilde{\eta}^{(q)}_{t+1}, \tilde{x}_{t+1}, \tilde{\pi}_{t+1})'$ contain the backward-looking variables. The variance-covariance matrix $Cov(w_t) = \Sigma_w$ in vectorized form is given by

$$vec(\Sigma_w) = (I_{(q+3)^2} - M \otimes M)^{-1} vec(gg')\sigma^2 \tag{D.7}$$

where $M = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}$. According to Söderlind (1999) Z_{11}, S_{11}, and T_{11} follow from the Generalized Schur decomposition $\Phi = \overline{Q}'S\overline{Z}'$ and $\Psi = \overline{Q}'T\overline{Z}'$ with

$$S = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix}, \quad T = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix}, \quad Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \tag{D.8}$$

$\overline{Q}$ and $\overline{Z}$ are the complex-conjugates of Q and Z, respectively. The $(q+3 \times q+3)$-dimensional matrices S_{11} and T_{11} contain the stable eigenvalues of the system (D.1).