

# Modeling Consumer Confidence and its Role for Expectation Formation: A Horse Race

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## Abstract

The notion of bounded rationality has received a considerable attention in the midst of debate over the usefulness of various macroeconomic models. In this paper we empirically seek to analyze the baseline New-Keynesian model with heterogeneous agents who may adopt various heuristics used to forecast future movements in consumption. In the spirit of Carl Chiarella, we consider various approaches of behavioral heterogeneity in macroeconomic models. Agents could exhibit an optimistic or pessimistic view or act as fundamentalists or chartists when forming expectations on future consumption based on discrete choice. Our empirical results via the Simulated Method of Moment Approach show that consumer confidence in the US is heavily grounded on consumers' emotional state (with respect to optimism and pessimism), while for the Euro Area it is most likely technical in nature (with respect to fundamentalists and chartists). These heuristics lead to an equivalent or even better fit to the data compared to the hybrid version of the baseline New-Keynesian model. We argue that this study could open up new possibilities for estimating bounded rationality models and policy analysis.

**Keywords:** Bounded Rationality; Consumer Confidence; New-Keynesian Model; Forecast Heuristics; Simulated Method of Moments.

**JEL classification:** C53, D83, E12, E32.

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# 1 Introduction

Over the last decade a plethora of studies has addressed the validation of the rational expectation paradigm in the midst of debate over the usefulness of various macroeconomic models. In this regard the notion of bounded rationality becomes increasingly important in the significance of agent's expectation formation process over the business cycle. Since the burst of the bubble in the U.S. housing market in late 2007, macroeconomists have been aware of the design of optimal policy strategies in circumstances where agents do not behave rationally. With respect to expectation formation on a macro level, changes in consumer confidence could have a significant impact on economic development. This view (while being already addressed by various authors before) was boosted by Akerlof and Shiller who analyzed the mechanism behind the global financial crises at its early stages. To quote from their seminal book on 'Animal Spirits' [Akerlof and Shiller (2009), p. 12]:

Economists have only partly captured what is meant by trust or belief. Their view suggests that confidence is rational: people use the information at hand to make rational predictions; they then make a rational decision based on those rational predictions. Certainly people often do make decisions, confidently, in this way. But there is more to the notion of *confidence*. The very meaning of trust is that what goes beyond the rational. *Indeed the truly trusting person often discards or discounts certain information.* She may not even process the information that is available to her rationality; even if she has *processed* it rationally, she still may not *act* on it rationally. She acts according to what she trusts to be true.

In this regard, economic confidence results from the waves of optimism and pessimism, which reflects animal spirits as imposed by Keynes (1936). As a key characteristic of this concept with respect to decision making, agents do behave bounded-rational rather than purely rational. In the latter case agents forecast future economic dynamics given full information of the underlying structure of the economy and properties of the (exogenous) shocks given. Instead, as mentioned by Akerlof and Shiller above, certain information could be discarded while emotional states play a major role for expectation formation. Similarly, agents might not be affected by those emotional states but be aware of the fact that information is not fully available. In this case, past information and/or fundamental values processed via backward-looking rule-of-thumb behavior (i.e. heuristics) serve as a promising approach for the development of future expectations. Following this argumentation gives rise to the formulation of so-called bounded rationality models which account for the behavior of heterogenous agents.

The concept of animal spirits is not limited to describing investment decisions (as primarily stated by Keynes) but, of course, also to explaining the

determination of consumption. In this paper we focus on the modeling and evaluation of heuristics which display the formation of *consumer confidence*. It is well known empirically that consumption is the main driver of GDP. This stylized fact puts the following question into the spotlight: how a change in confidence affects heterogeneous consumers' decision making process and, hence, macroeconomic dynamics? In this paper, we seek to find out which kind of forecast consumer heuristics being considered could lead to the best description of a standard macroeconomic model to empirical data. Most importantly, we investigate whether the expectation formation of bounded rational agents could outperform purely rational expectations. Indeed previous studies show ambiguous results about expectation formation processes and give the impression that the final picture is not conclusive yet.

In an early work on animal spirits, Blanchard (1993) examines empirically the extent to which consumer confidence reflects a negative consumption shock in the 1990–1991 recession in the US. He finds evidence for agents with a limited information processing ability. The author suggests that perfect foresight does not fully explain the drop in consumers' expectation, which allows some scope for an interpretation of confidence as a driver of agents' decision making process. This view, however, is questioned by Barsky and Sims (2012). They examine the impact of news shocks and only a noise-ridden signal of that kind of shock, where the latter can be regarded as a type of animal spirits. According to their empirical results, animal spirits only could contribute weakly to the observed relationship between confidence and economic activity. Similarly, Ludvigson (2004) provides an overview of studies on the relationship between consumer confidence and economic activity. He reports mixed results with respect to the question on whether consumer surveys are connected to precautionary saving motives based on the literature. The author also states that while specific survey measures are designed to reflect predicted future outcome of consumer expenditure growth, the same amount of information can be found in various economic and financial indicators. From a theoretical point of view, Gomes (2010) addresses the need of animal spirits in a non-linear endogenous growth model in order to describe business cycle fluctuations. The author shows that the economic system undergoes a bifurcation at specific conditions when agents makes economic decisions today based on past information on the future path of the economy.

Indeed, the modeling of expectation formation and, especially, the estimation of bounded rationality models in the absence of the rational paradigm stands out as a so far novel branch of the macroeconomic literature. It is noteworthy that important theoretical and empirical contributions with respect to models with focus on heterogeneous agents have been already investigated in great detail by financial economists. Pioneers in the field are without a doubt Carl Chiarella along with his co-authors. For example, Chiarella et al. (2010) estimate a capital asset pricing model with fundamentalists, trend followers and noise traders. They show that a systematic change in the market portfolio, asset prices and returns rely on the changes in investor perceptions. The

behavior of investors includes the trend extrapolation of the chartists, the mean-reversion of the fundamentalists, and the strength of the noise traders. Further, Chiarella et al. (2011) evaluate a Markov-switching model with fundamentalists and chartists and show that their model succeeds in matching the boom and bust periods in the US stock market from 2000 to 2010. In an earlier contribution, Asada et al. (2007) estimate a dynamic three-dimensional AD-AS model via the Generalized Method of Moments (an approach which is closely related to ours in this paper) and use the parameter estimates for the calibration of a small-scale New-Keynesian model.

In the spirit of Carl Chiarella, we modify a bounded-rational model, i.e. under consideration of rule-of-thumb possessing agents, and estimate the model parameters via the Simulated Method of Moments. These kinds of evolutionary learning approaches have been widely used to model the expectation formation process in dynamic stochastic general equilibrium (DSGE) models. This work has been pioneered by the work of Brock and Hommes (1997), as well as Gaunersdorfer et al. (2008), among others. Agents sort themselves into different categories related to both their emotional states and specific professional forecasting rules based on heuristics. As an prominent example, De Grauwe (2011) incorporates this kind of mechanism into the New-Keynesian framework where economic agents are, in fact, either bounded-rational or exhibit perfect rational expectations. The empirical evidence for this model specification is, so far, ambiguous. Jang and Sacht (2016) find that the bounded-rational version of the New-Keynesian model for the Euro Area exhibits an equivalent good fit to the data as the one with rational expectations being assumed. Liu and Minford (2014) show evidence for the opposite. Therefore the analysis of the impact of the bounded-rational behavior of agents is not a trivial one. In this paper we try to judge the validity of specific (competing) rule-of-thumb behavior in expectations formation processes. To show this, we define a *horse race* as the evaluation of the performance of different heuristics via a well-defined objective function. In particular, we seek to find specific pair of heuristics which leads to the smallest deviation of the model empirical second moments from their empirical counterparts. A similar purely theoretical investigation with respect to solution methods of DSGE models under rational expectations is presented by Anderson (2008).

It is a well-known fact that inertia in the dynamics of consumption/output and inflation is observed empirically. Without exogenous persistence assumed, various authors show that the only shock-driven forward-looking New-Keynesian model under rational expectations is not able to reproduce equivalent impulse response functions (cf. Chari et al. (2002) and Christiano et al. (2005)). As we will discuss in this paper, consumer confidence is the main driver of highly persistent dynamics in consumption. For this case, our analysis is able to examine what kind of bounded-rational models stand out as alternatives if the rational expectations counterpart performs poorly. Various attempts in the literature modify the baseline New-Keynesian model in order to account for persistence in empirical data. The result is the so-called hybrid version of the model under

rational expectations with leads and lags in the dynamics IS equation and the New-Keynesian Phillips curve. The formulation of the backward-looking parts being added are basically ad hoc (cf. Smets and Wouters (2005) and Christiano et al. (2005) among others). In this paper, our investigation reveals that from an empirical point of view, the bounded rational model specifications lead to an equal or even better fit to empirical data while they offer more detailed information on expectation formation schemes. Our contribution is closely related to Cornea-Madeira et al. (2017) who provide empirical evidence for behavioral heterogeneity in US inflation dynamics based on two different group of fundamentalists and chartists who adopt simple rule-of-thumb behavior. Furthermore, their results reveal that endogenous switching between both groups depends on the realization of agents' performance when predicting future outcomes. In comparison to their study, in this paper we focus on the consumer side of the US economy and the Euro Area rather than inflation dynamics, i.e. we consider different types of heuristics regarding consumption expectations and only one bounded-rational specification for the New-Keynesian Phillips curve.<sup>1</sup>

The paper is structured as follows. Section 2 examines the role of consumer confidence in the US and the Euro Area based on descriptive statistics. Section 3 develops several forecast heuristics applied in a heterogeneous agent-based version of the New-Keynesian model (NKM henceforth) under consideration of the discrete choice mechanism. The parameter estimates of the various model specifications are evaluated according to their fitness criteria in Section 4. In Section 5 we shed a light on the need for hybridity in the corresponding version of the New-Keynesian model under rational expectations based on the evaluation of its bounded-rational counterparts. Finally, Section 6 concludes. The Simulated Method of Moments (SMM henceforth) approach is described in Appendix A1 as the empirical estimation method of our choice introduced by Franke et al. (2015). The impulse response functions in case of a supply and a monetary policy shock (linked to our discussion in Section 5) are relegated to the Appendix A2.

## 2 Consumer Confidence and the Business Cycle

The main concern of business cycle analysis is to find the reason why the economy is constantly going through periods of booms and busts. In general, the question arises of how business cycles come about. In theory the answer is simply given by the decomposition of the Gross Domestic Product (GDP henceforth) into its main components of private consumption, aggregate investment and the trade surplus based on the system of national accounts. Empirically, private consumption contributes the most to the level of GDP. More precisely, according to recent data on total household spending provided by the OECD (2016a), private consumption amounts to around 68% and 56% relative to US

<sup>1</sup>We refer to Cornea-Madeira et al. (2017) for a detailed overview of the literature on mixed evidence about the role of forward-looking and backward-looking components in DSGE models.

and Euro Area GDP, respectively, excluding housing and government transfers. By including the latter one only, these numbers range even around 75% and 69% for the US and the Euro Area, respectively.

Indeed, consumer confidence plays a crucial role in the determination of household spending. Empirical evidence implies that there is a significant relationship between consumer confidence, household spending and GDP. In Figure 1 we compare the annual growth rates of household spending (OECD (2016a)) to the consumer confidence index (CCI for short, OECD (2016b)) and GDP (OECD (2016c)) in the US and the Euro Area. The times series of the CCI consists on monthly data, which is obtained through the calculation of annual average values and, hence, the corresponding growth rates.<sup>2</sup> The time spans cover the period from 1971 to 2014 for the US, but our focus on the Euro Area data is limited to a short period of time from 1997 to 2014 due to partial data availability.

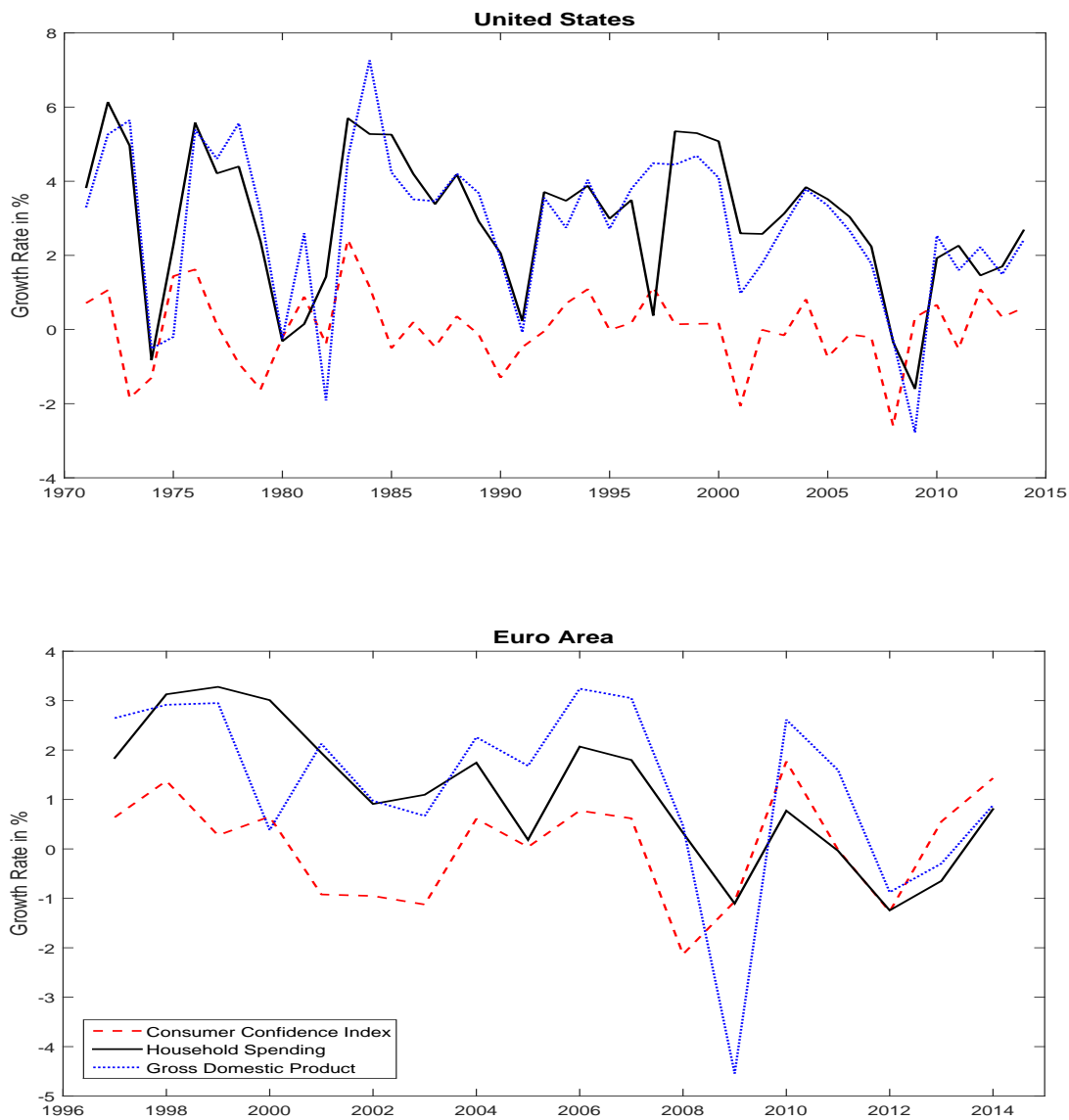
The upper panel of Figure 1 below shows the development of these indicators for the US economy. Two observations are worth mentioning. First, the time series move closely together, i.e. peaks and troughs occur at almost the same points in time. This holds especially for the growth rates of household spending and GDP. In this regard, our second observation is concerned with the lag behavior of the CCI. This suggests that ups and downs of the other two indicators follow the ones of the CCI. This is not surprising as the CCI can be identified as a leading indicator, used by the US federal government and firms to determine economic policy and business decisions, respectively, in order to respond to predicted developments in private consumption. Indeed the co-movements in all indicators become apparent especially in times of economic slowdowns (given by very low or even negative growth rates) like e.g. both oil crises in 1973 and 1979, at the beginning of the Great Moderation period around 1980, the burst of the dot-com bubble in 2001 and most clearly in case of the burst of the US housing bubble in 2007. We can make similar comments on the observations for the Euro Area. The corresponding time series are shown in the lower panel of Figure 1. In comparison, however, the time series behave more in a synchronous way (i.e., one-to-one overlapping peaks and troughs). For example, this holds for 2004 and 2007, as well as at the peak of the sovereign debt crises during the period from 2010 to 2012.

Overall, our descriptive analysis suggests a strong relationship between consumer confidence, household spending and their pass-through to GDP fluctua-

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<sup>2</sup>The CCI is provided on a monthly basis by ‘The Conference Board’ which is an US non-profit business membership and research group organization. A survey of 5,000 households in the US and Euro Area, respectively, consists on five questions each related to current and future business and employment conditions as well as the prediction of future total family income. Answers can be ‘positive’, ‘neutral’ or ‘negative’. The index values are calculated based on the relation of each question’s positive responses to the sum of its positive and negative responses. For more information the interested reader may refer to the organizations web page via <https://www.conference-board.org>.

**Figure 1:** Growth rates of the consumer confidence index, total household spending and the Gross Domestic Product for the US and the Euro Area

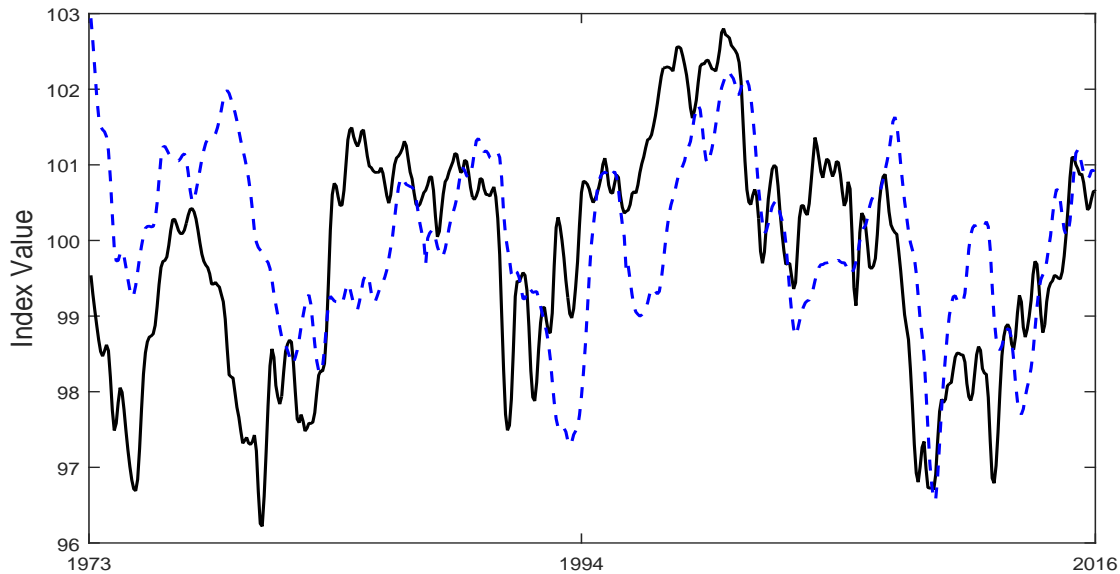


Source: OECD, authors' own calculations.

Note: The solid, dashed and dotted lines depict the growth rates of total household spending, the consumer confidence index and the Gross Domestic Product (GDP), respectively. The growth rate of the consumer confidence index is calculated on average based on monthly data. The underlying monthly series is amplitude adjusted with a long term average of 100. Household spending is measured on an annual growth rate and GDP growth in percentage change from the previous quarter and from the same quarter of the previous year, respectively.

tions. This view is not entirely new. For example, Milani (2014) uses a bounded rationality model close to Jang and Sacht (2016) and shows that so-called sentiment shocks (in terms of shifts from optimistic to pessimistic expectations and vice versa) account for roughly 40% of GDP fluctuations in the US economy. Golinelli and Parigi (2004) focus on the forecast performance of consumer sentiments. They show that a restricted VAR model with consumer sentiments outperform an unrestricted model without sentiments. Similarly, Sacht (2015) points out that the recovery of the Spanish economy since 2014 is mainly grounded on the reversal in consumers confidence from pessimistic to optimistic view on future economic developments. This list is not conclusive. In general, the impact of consumer confidence (or sentiments) on economic fluctuations can be measured in a quantitative way e.g. either by applying Granger-causality tests, in-sample forecast exercises and/or vector decomposition methods (for the latter see e.g. van Aarle and Kappler (2012)).

**Figure 2:** Development of the CCI in monthly magnitudes for the US and the Euro Area (1973:M1-2016:M1)



*Source:* OECD

*Note:* The *solid* and *dashed* lines depict the index values of the CCI in monthly magnitudes for the *US* and the *Euro Area*, respectively. For clear arrangement, on the x-axis we indicate only the years 1973, 1994 and 2016, which are recorded as the corresponding values in January.

However, it is not trivial to understand the psychological concepts behind the establishment of confidence as a driver of real economic fluctuations. This holds especially with respect to a theoretical foundation. In this paper we try to fill the gap between empirical observations and model based expectation formation. To begin with, we will have a closer look at the degree of autocorrelation



of the CCI time series. Our interest for this property manifests in the following statement: consumer confidence exhibits a high degree of persistence, which contributes to an increase in confidence itself according to a backward-looking expectation formation scheme. In addition, the latter then should account for the high degree of inertia in the time series of output.

Figure 2 above depicts the development of the CCI for the US and the Euro Area. The data are given on a monthly magnitude (OECD (2016b)). Since data for the Euro Area is only available from January 1973 onwards, we focus on the time period starting at this date until January 2016.<sup>3</sup> As the amplitude of times series is adjusted with a long term average of 100, index values above (below) that value indicate an increase (or decline) in consumer confidence in future economic developments. Figure 2 clearly shows that the index values fluctuate around the long term trend. Most importantly, the degree of autocorrelation at the first lag is equal to 0.988 and 0.983 for the US and the Euro Area, respectively. This finding strengthens our view that consumer confidence is highly persistent over time. Based on our observations in Figure 1, the degree of autocorrelation in the CCI *growth rate* amounts to 0.753 (for the US) and 0.552 (for the Euro Area). These lower numbers relative to the ones obtained by monthly data are, of course, explained by the lost of information as we calculate the growth rates on average on an annual magnitude in the former case. Nevertheless, the persistence in the CCI growth rates contributes to the (moderate) degree of autocorrelation in the growth rates of household spending given by 0.412 (for the US) and 0.639 (for the Euro Area). This observation supports our statement about a strong relationship between consumer confidence and private consumption expenditure. In the following Section we seek to interpret this empirical evidence by considering decision rules, i.e. heuristics which account for the aspects of backward-looking expectation formation.

### 3 Expectation Formation in the Baseline NKM

The baseline NKM reads as follows:

$$c_t = \tilde{E}_t^j c_{t+1} - \tau(r_t - \tilde{E}_t^j \pi_{t+1}) + \varepsilon_{c,t} \quad (1)$$

$$\pi_t = \nu \tilde{E}_t^j \pi_{t+1} + \kappa c_t + \varepsilon_{\pi,t} \quad (2)$$

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)(\phi_\pi \pi_t + \phi_c c_t) + \varepsilon_{r,t} \quad (3)$$

$$c_t = y_t \quad (4)$$

where the superscript  $j = \{\text{RE}, \text{BR}\}$  refers to the rational expectations (RE) and the bounded rationality (BR) model, respectively. The corresponding expectations operator is  $\tilde{E}_t^j$ , which has to be specified for both models. It goes without saying that all variables are given in quarterly magnitudes.

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<sup>3</sup>Note that our own calculation of the CCI growth rates shown in Figure 1 are based on the monthly data underlying Figure 2. However, the time span being considered in Figure 1 differs due to the fact that information on household spending and GDP growth provided by the OECD are not available before 1971 (1997 for the Euro Area) and after 2014.

In equation (1), private consumption expenditure stems from intertemporal optimization of consumption and saving, which leads to consumption smoothing (based on the realizations of the real interest gap denoted by  $r_t - \tilde{E}_t^j \pi_{t+1}$ ). The parameter  $\tau \geq 0$  denotes the inverse intertemporal elasticity of substitution in consumption behavior. Equation (2) represents the New-Keynesian Phillips Curve (NKPC henceforth) where aggregate consumption ( $c_t$ ) acts as the driving force of inflation ( $\pi_t$ ) dynamics due to monopolistic competition and the Calvo-type sticky price setting scheme. The slope of the NKPC is given by the parameter  $\kappa \geq 0$ .  $\nu$  measures the discount factor ( $0 < \nu < 1$ ). According to the Taylor rule with interest rate smoothing (equation (3)), the nominal interest gap ( $r_t$ ) is a predetermined variable with the corresponding persistence parameter  $0 \leq \phi_r \leq 1$ . The monetary authority reacts directly to contemporaneous movements in consumption ( $\phi_c \geq 0$ ) and inflation ( $\phi_\pi \geq 0$ ). We assume that the exogenous driving forces in the model variables follow idiosyncratic shocks  $\varepsilon_{s,t}$ , which are independent and identically distributed around mean zero and variance  $\sigma_s^2$  with variables  $s = \{c, \pi, r\}$ .

Note that we focus on the forward-looking NKM to avoid a bias in the results stemming from the competitive approaches of backward-looking expectation formation schemes (as introduced below) and the model's intrinsic persistence (according to e.g. consumption habits and the price indexation scheme). Hence, the RE model type is purely forward-looking while the BR one is purely backward-looking. In Section 6, we consider the hybrid rational expectations model and report the model parameter estimates. This can be used to provide a comparison to bounded rationality models and to show that the backward-looking elements in the hybrid RE model can be most likely explained by bounded-rational heuristics. Most importantly, as a main characteristic of the DSGE model it is required that consumption expenditure equals output in equilibrium. Hence, equation (4) implies that equation (1) expresses nothing else than the standard dynamic IS curve.<sup>4</sup> All variables are given in gap notation, where we consider the deviation of the contemporaneous realization of this variable from its steady state value denoted by  $\bar{s} = \{\bar{c}, \bar{\pi}, \bar{i}\}$ . In the following we omit the expression 'gap' for a clear arrangement. We refer to Section 5 for more details regarding the implications for our estimations.

Under *rational expectations* (RE), the forward-looking terms are described by the expectations with respect to consumption and inflation at time  $t + 1$  in the equations (1) and (2):

$$\tilde{E}_t^{RE} z_{t+1} = E_t z_{t+1} \quad (5)$$

with  $z = \{c, \pi\}$  and where  $E_t$  denotes the expectation operator conditional on

<sup>4</sup>While it is common in the macroeconomic literature on DSGE models to express the dynamic IS equation (1) in terms of the output gap  $y_t$ , we rely on the notation for private consumption given by  $c_t$  instead. The reason is that we want to avoid any confusion for the reader as we focus on consumption and, in particular, consumer confidence. Note that the appearance of  $c_t$  in the NKPC (2) and the Taylor rule (3) are justified from a theoretical point of view under consideration of the equilibrium condition (see Galí (2015) for more details).

information at time  $t$ . According to the RE hypothesis, agents act under perfect foresight, where the future paths of both, in this case, jump variables are known.

With respect to the *bounded rational* (BR) version of the model, we distinguish between expectation formation with respect to consumption and inflation, respectively. For the BR model, we apply specific heuristics adopted by Gaunersdorfer et al. (2008), Chiarella et al. (2009) and De Grauwe (2011). Regarding *consumption* expectations, agents are able to sort themselves into four groups of forecasters expressed through the following heuristics:

$$E_t^F c_{t+1} = \bar{c} + \psi_c(c_{t-1} - \bar{c}) \quad (6)$$

$$E_t^C c_{t+1} = c_{t-1} + \xi_c(c_{t-1} - c_{t-2}). \quad (7)$$

$$E_t^O c_{t+1} = \frac{1}{2} \cdot [\beta + \delta\lambda_{c,t}] \quad (8)$$

$$E_t^P c_{t+1} = -\frac{1}{2} \cdot [\beta + \delta\lambda_{c,t}] \quad (9)$$

where for the steady state value  $\bar{c} = 0$  holds for simplicity. This implies that the equations (6) to (9) reflect *consumers'* forecast heuristics.

In the absence of the RE hypothesis, two groups of the agents apply the forecasting rules (6) and (7). We refer to this heuristics simply as the rule-of-thumb behavior since they consist on backward-looking elements as described in the following. The *fundamentalists* (F) and *chartists* (C) account for professional forecast behavior which suggests the absence of the emotional states with limited information. Fundamentalists believe in a convergence of the future value(s) towards the steady state value  $\bar{c}$  with the speed of convergence given by  $0 \leq \psi_c \leq 1$ . A quick (slow) movement is observed in the case where  $\psi_c$  is close to 0(1). Chartists form their expectations based on historic patterns in the time series. Under consideration of the past realization and the relation between the first and second lag, this type of agents either extrapolate the last change in  $c$  ( $\xi_c > 0$ ) or expect a reversal instead ( $\xi_c < 0$ ). It can be said that these heuristics are *technical* in nature.

In addition, with respect to the equations (8) and (9), we follow directly the specifications by Jang and Sacht (2016) when modeling and estimating the divergence in belief. Here, we assume that agents may adopt either an *optimistic* or *pessimistic* (in the following indicated by the superscripts  $O$  and  $P$ , respectively) attitude towards movements in future consumption. Hence, both types of agents are uncertain about the future dynamics of consumption and therefore predict a subjective mean value of  $c_{t+1}$  measured by  $\beta \geq 0$ . However, this kind of subjective forecast is generally biased and therefore depends on the volatility in consumption, i.e. given by the unconditional standard deviation  $\lambda_{c,t} \geq 0$ . The corresponding parameter  $\delta \geq 0$  measures the degree of divergence in the movement of economic activity. We consider symmetry with respect to  $\beta$  and  $\delta$ : optimists expect that the consumption will differ positively from the steady state value  $\bar{c}$  given by the value of  $\beta/2$ , while pessimists will expect a negative deviation on the same magnitude. We refer to these heuristics as the

*emotional* ones.

Given these assumptions for the expectation formation process for consumption, we estimate the different scenarios, i.e. specifications of the model. Hence, the horse race consists on six scenarios, where the corresponding heuristics (5) to (9) are considered. In the first scenario the model is estimated based on the RE hypothesis according to equation (5) only. The second scenario focuses on the so-called *pure technical block* (PTB), i.e. the equations (6) and (7) hold. The third scenario consists on the so-called *pure emotional block* (PEB), where here the equations (8) and (9) are applied only. As a mixture of both specifications we introduce the *emotional-fundamental block* (EFB) and the *emotional-chartist block* (ECB) as the fourth and fifth scenario, respectively. The former makes use of the equations (6), (8) and (9), while the latter consists on (7) to (9) instead. Finally, the sixth scenario is labeled as the *cognitive aggregate block* (CAB), where we allow for the existence of all four groups of heterogenous agents.

These separations are useful in order to compare the fit of the model specifications with respect to the nature of expectation formation in spirit of our horse race exercise. In particular, the emotional blocks reflect animal spirits, i.e. the waves in optimistic and pessimistic beliefs, while according to the technical block the forecasting rules are applied in the absence of the emotional state. By considering these scenarios of heuristics we account for the observations taken from the previous section. Therefore we try to answer the following three questions: (i) which combinations of heuristics could account the most for the high degree of autocorrelation in consumers' confidence? (ii) Which specification of the model could lead to the best description of the data? (iii) Does a BR model based on heuristics alone accounts for inertia in the time series or is a structural representation of a RE model with leads and lags required? We address the latter question in Section 5.

Under BR, the switching from one group to the other group is based on discrete choice theory and is described as follows. The expression for the *market forecast* regarding consumption across the four groups is given by

$$\tilde{E}_t^{BR} c_{t+1} = \sum_{i=1}^4 (\alpha_{c,t}^{k\{i\}} \cdot E_t^{k\{i\}} c_{t+1}) \quad (10)$$

with  $k = \{O, P, F, C\}$ . The probability  $\alpha_{c,t}^k$  indicates a stochastic behavior of the agents who adopt a particular forecasting rule (i.e. out of the equations (6) to (9)). More precisely,  $\alpha_{c,t}^k$  can be interpreted as the probability being an optimist, pessimist, fundamentalist or chartist with respect to development of consumption in period  $t$ . The selection of the forecasting rules (6) to (9) depends on the *forecast performances* of each group  $U_t^k$  given by the mean squared forecasting error. The utility for the forecast performances can be simply updated in every period as (cf. Brock and Hommes (1997))

$$U_{c,t}^k = \rho U_{t-1}^k - (1 - \rho)(E_{t-2}^k c_{t-1} - c_{t-1})^2, \quad (11)$$

where the parameter  $\rho$  is used to measure symmetrically the memory of the four different types of agents ( $0 \leq \rho \leq 1$ ). Here  $\rho = 0$  suggests that agents have no memory of past observations, while  $\rho = 1$  means that they have infinite memory instead. Agents can revise their expectations by applying the discrete choice approach under consideration of their forecast performances. The different types of performance measures can be utilized for  $\alpha_{c,t}^k$  as follows:

$$\alpha_{c,t}^{\tilde{k}} = \frac{\exp(\gamma U_t^{\tilde{k}})}{\sum_{i=1}^4 \exp(\gamma U_t^{k\{i\}})}; \quad (12)$$

$$\alpha_{c,t}^C = \frac{\exp(\gamma U_t^C)}{\sum_{i=1}^4 \exp(\gamma U_t^{k\{i\}})} \cdot \exp\left[-\frac{(c_{t-1} - \bar{c})^2}{\varpi}\right], \quad (13)$$

with  $\varpi > 0$  being the correction term. The parameter  $\gamma \geq 0$  denotes the intensity of choice. It goes without saying that the equations (10) to (13) have to be adjusted conditional on the expectation formation scenario being considered. It is worthwhile to mention that we distinguish the probabilities of the subgroups  $\tilde{k} = \{O, P\}$  in (12) from the one regarding the chartists  $C$  in (13). We consider this specific probability to account for a decline in the fraction of chartists as consumption deviates further from its steady state value, where ‘speculative bubbles’ cannot last forever (cf. Gaunersdorfer et al. (2008)). According to Hommes (2001), the last term in equation (13) denotes a transversality condition in the model with heterogenous agents and can be seen as a penalty term in the chartists’ forecast performance. Of course, the probability for being a fundamentalist is then given by

$$\alpha_{c,t}^F = 1 - \sum_{i=1}^3 \alpha_{c,t}^{\tilde{k}\{i\}} \quad (14)$$

with  $\tilde{k} = \{O, P, C\}$ . Again, according to the different scenarios considered, the specification in equation (14) will differ accordingly.

The steady state value for consumption ( $\bar{c}$ ) cannot be seen as a target rate but a (constant) trend in economic activity with respect to business cycle fluctuations instead. On the contrary, the central bank seeks to stabilize inflation via the interest channel of monetary policy. In particular, the monetary authority anchors expectations by announcing a target for inflation given by  $\bar{\pi}$ . The inflation fundamentalists consider this pre-commitment strategy to be fully credible. The corresponding forecasting rule becomes then

$$E_t^F \pi_{t+1} = \bar{\pi} \quad (15)$$

with a target rate of  $\bar{\pi} = 0$  for simplicity. The inflation chartists expect that the future value of the inflation gap is given by

$$E_t^C \pi_{t+1} = \pi_{t-1}. \quad (16)$$

Hence, we adopt the same heuristics with respect to fundamentalists and chartists as before (see equations (6) and (7) above) but with  $\psi_\pi = \xi_\pi = 0$  instead. We

**Table 1:** The design of the horse race: RE & BR Scenarios

Scenario	Heuristics w.r.t. $y_t$	$\alpha_{c,0(-)}^0$	$\alpha_{c,0(-)}^P$	$\alpha_{c,0(-)}^F$	$\alpha_{c,0(-)}^C$	Set of Information w.r.t. $\tilde{E}_t^j c_{t+1}$
RE	(5)	0	0	0	0	$S_1(c_{t+1})$
PTB	(6),(7)	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$S_2(c_{t-1}, c_{t-2}, \bar{c})$
PEB	(8),(9)	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$S_3(\lambda_{c,t})$
EFB	(6),(8),(9)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$S_4(\lambda_{c,t}, c_{t-1}, \bar{c})$
ECB	(7),(8),(9)	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$S_5(\lambda_{c,t}, c_{t-1}, c_{t-2})$
CAB	(6),(7),(8),(9)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$S_6(\lambda_{c,t}, c_{t-1}, c_{t-2}, \bar{c})$

put these constraints on the heuristics to consider the impact of consumer confidence in isolation while following the description of the so-called inflation targeters and extrapolators imposed by De Grauwe (2011). This scheme also applies with respect to the expectation on future house prices and consumption of non-durable goods as presented in Bofinger et al. (2013), where they assume that the inflation fundamentalists apply a forecast rule equal to the steady state value of zero. It goes without saying that the equations (10) to (14) have to be adjusted in case of the inflation expectation formation process. Note that the memory parameter given by  $\rho$  remains the same for consumption and inflation. Further, while we vary the heuristics with respect to consumption expectations, the ones for the inflation gap are always the same and given by (15) and (16) under BR.

Table 1 above shows the design of the horse race. The structure of the baseline NKM is given by the equations (1) to (4). For the six different scenarios (one for RE and five for BR) we make use of the heuristics regarding consumption according to the equations mentioned in the second column and in the text. Expectations in the RE case are determined by equation (5). For the BR model specifications, the equations (10) to (14) are used to simulate consumption and inflation. Note that some heuristics for  $c_t$  are ruled out according to the scenario considered. This is also mimicked by the zero entries in the third to sixth column which display the fractions of the different groups of agents  $\alpha_{c,\tilde{t}}^k$  at the beginning of the estimation. In this regard period  $\tilde{t} = 0(-)$  indicates the point in time before the shocks will hit the economy. The entries, which are not set to zero, indicate a uniform distribution of fractions a priori. The last column contains the agents' information sets (S) at time  $t$  used for their forecasts. For completeness, the latter depends also on the fractions  $\alpha_{c,t}^k$  which are computed at the beginning of each period. Again, the heuristics regarding inflation remain unchanged and are given by (15) and (16) together with  $U_{\pi,t}^F$ ,  $U_{\pi,t}^C$ ,  $\alpha_{\pi,t}^F$  and  $\alpha_{\pi,t}^C$ .

In the Appendix A1, we describe the SMM approach to estimating the *structural* ( $\tau, \kappa, \phi_r, \phi_\pi, \phi_c$ ) and the *bounded rationality* ( $\beta, \delta, \psi_c, \xi_c$ ) parameters

as well as the corresponding *standard deviations* ( $\sigma_c, \sigma_\pi, \sigma_r$ ) of the model specifications. In Section 5 we are going to estimate also the additional parameters for habit formation ( $\chi$ ) and price indexation ( $\alpha$ ) as part of the hybrid version of the NKM.<sup>5</sup>

## 4 Empirical Application

### 4.1 Data

The US data set is taken from the web page of the Federal Reserve Bank of St. Louis (<https://fred.stlouisfed.org>). The sample spans over the period from 1975:Q1 to 2009:Q4. Inflation is measured using the seasonally adjusted consumer price index with 2009 as the base year. Output is obtained from seasonally adjusted real GDP based on billions of chained 2009 dollars. The effective federal funds rate is used to measure the short-term nominal interest rate in the US.

We retrieve the Euro Area data set from the 10th update of the Area-Wide Model quarterly database (<http://www.eabcn.org/page/area-wide-model>; see Fagan et al. (2001)). To be consistent with the time span for the US economy, the sample covers the periods 1975:Q1 to 2009:Q4. The consumption deflator is used to measure inflation in the Euro Area. The short-term nominal interest rate and real GDP are used to measure the gaps in the nominal interest rate and output in the Euro Area.<sup>6</sup> A standard smoothing parameter of  $\lambda = 1600$  is used to estimate the trend of the observed data from the Hodrick-Prescott filter for output, inflation and the nominal interest rate. Note that according to the equilibrium condition  $c_t = y_t$  we consider the output gap time series as proxy for private consumption (due to the limited data availability of the latter) within our empirical analysis. In the estimation procedure, we take the model to the second moment conditions derived from the gaps based on the data set.

### 4.2 Results

The parameter estimates from the *US economy* for all six scenarios are given in Table 2 below. The values of the quadratic objective function  $J$  can be found in the next-to-last row.  $J$  measures the degree of matching the simulated time series to the empirical ones (according to equation (24) to be found in the

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<sup>5</sup>Across all estimations, the parameters of the discount factor  $\nu$  and the memory parameter  $\rho$  are calibrated to 0.99 and zero, respectively. In the former case, we simply follow the literature, where in an overwhelming majority of studies the same value for  $\nu$  is the result of an estimation. In the latter case, we correspond to our results, which show that this parameter is insignificant across all scenarios being estimated here. These results are available upon request. Furthermore, we follow Gaunersdorfer and Hommes (2007) and assume that the correction term  $\varpi$  is equal to 1800.

<sup>6</sup>The time series in the Area-Wide Model database have the following abbreviations: consumption deflation = ‘PCD’, short-term nominal interest rate = ‘STN’ and real GDP = ‘YER’.

Appendix A1). This suggests that the lower the value of  $J$ , the better the fit of model to the data will be.

**Table 2:** Estimation results based on US data (RE & BR scenarios)

Label	RE	CAB	PEB	PTB	EFB	ECB
$\tau$	0.000 -	0.128 0.000 - 0.285	0.369 0.000 - 1.370	0.184 0.000 - 0.395	0.371 0.222 - 0.520	0.321 0.000 - 0.675
$\sigma_c$	1.734 1.397 - 2.071	0.287 0.000 - 0.626	0.556 0.000 - 1.351	0.408 0.0 - 0.895	0.543 0.267 - 0.818	0.378 0.000 - 0.833
$\kappa$	0.332 0.112 - 0.552	0.205 0.149 - 0.261	0.216 0.130 - 0.302	0.185 0.140 - 0.229	0.213 0.175 - 0.252	0.243 0.000 - 0.527
$\sigma_\pi$	1.605 1.156 - 2.055	0.253 0.000 - 0.507	0.261 0.060 - 0.461	0.206 0.000 - 0.520	0.240 0.018 - 0.461	0.212 0.000 - 0.432
$\phi_\pi$	1.468 1.000 - 2.848	2.051 1.369 - 2.733	1.918 1.000 - 3.046	2.274 1.295 - 3.254	1.914 1.080 - 2.747	1.946 1.318 - 2.574
$\phi_c$	2.041 0.643 - 3.439	0.363 0.000 - 0.903	0.417 0.000 - 1.157	0.589 0.000 - 1.630	0.709 0.011 - 1.407	0.299 0.000 - 0.867
$\phi_r$	0.812 0.735 - 0.888	0.719 0.478 - 0.959	0.570 0.078 - 1.000	0.781 0.637 - 0.924	0.808 0.660 - 0.956	0.644 0.319 - 0.970
$\sigma_r$	0.002 -	0.358 0.000 - 0.862	0.314 0.000 - 0.850	0.221 0.000 - 0.576	0.151 0.000 - 0.417	0.304 0.000 - 0.965
$\beta$	-	3.220 1.474 - 4.967	2.253 0.878 - 3.628	-	3.282 1.598 - 4.967	2.527 0.809 - 4.244
$\delta$	-	0.635 0.000 - 1.755	0.577 0.000 - 1.445	-	0.531 0.000 - 1.550	0.775 0.000 - 2.024
$\psi_c$	-	0.737 0.534 - 0.940	-	0.897 0.564 - 1.229	0.951 0.657 - 1.0	-
$\xi_c$	-	1.373 0.367 - 2.419	-	0.758 -0.362 - 1.878	-	0.435 -0.798 - 1.668
$J$	213.53	42.47	212.40	39.28	43.29	61.28
$p$	0.000	0.989	0.000	0.998	0.989	0.674

*Note:* We use 78 moments (two years), based on the SMM approach. The 95% confidence intervals are given with a smaller size. The value of the objective function and the p-value are denoted by  $J$  and  $p$ , respectively. The degrees of freedom for  $\chi^2$  distribution amount to 66, 67, 68 (horse race) and 70 (RE, pure[ly forward-looking] BR). The 5% critical values for 66, 67, 68, 70 degrees of freedom are 85.96, 87.11, 88.25 and 90.53, respectively. No memory is assumed in the BR scenarios ( $\rho = 0$ ). The discount factor  $\nu$  is calibrated to 0.99. We set  $\varpi$  equal to 1800.

In the first step we compare the fitness of the model under the scenario RE to the remaining five ones. It can be seen that the model under rational expectations (with respect to both consumption and inflation) has the highest value given by  $J = 213.53$  compared to its bounded rationality counterparts. Interestingly, this holds except for the pure emotional block (PEB), where only optimistic and pessimistic consumer expectation formation are assumed. More precisely, the model under PEB with  $J = 212.40$  exhibits an equal good (or, let's say, worse) fit to the data as in the RE case. This is also confirmed by comparing the corresponding  $p$  values in both cases which are equal to zero. This observation comes on no surprise as the purely forward-looking NKM under rational expectations is not able to capture the high degree of inertia in



consumption/output and inflation dynamics. However, the difference in the matching performance under the RE scenario in relation to all remaining BR ones is quite remarkable. The question now arises: which kind of BR model specification could lead to the best possible description of the data?

Therefore, in the second step, we continue the discussion of the scenario which exhibits the best fit in terms of  $J$ . We identify the emotional-fundamental block (EFB) with  $J = 43.29$  as the most promising scenario to be assumed for the specification of the model to hold. Note here that consumers sort themselves into the groups of optimists, pessimists and fundamentalists, i.e. while their expectation formation process is heavily grounded on their emotional state, one-third of the whole population (a priori) prefer a rule-of-thumb with respect to the fundamental value of  $c$ , i.e. consumption in the steady state, instead. It is worth mentioning that the scenarios CAB and PTB provide almost the same quality fit to the data. This is based on the values for  $J$  and  $p$ , which are close to the ones under EFB. However, most of estimated parameters in both alternative scenarios are insignificant. In general, a model specification, where most of the parameters (especially the bounded-rational ones) are estimated to be insignificant, does not provide novel insights regarding practical use in policy analysis. Hence, we consider the scenario of EFB as our favourite choice across the bounded-rational specifications.

We now take a close look at the corresponding parameter estimates in the EFB scenario. First, the estimated values for  $\sigma_r$  and  $\delta$  are insignificant. Hence, we cannot rule out the possibility that the variance in the nominal interest rate shock, as well as the degree of divergence, are equal to zero. In the former case, the federal reserve monetary policy strategy does not rely on exogenous disturbances. In the latter case, consumers' subjective forecast may become unbiased with respect to the unconditional standard deviation of consumption expenditure. The pass-through of changes in the real interest rate on consumption dynamics (measured by  $\tau$ ) and consumption on inflation dynamics (measured by  $\kappa$ ) turns out to be 0.371 and 0.213, respectively. These values are higher than those reported in the majority of studies which investigate the estimation of models under rational expectations (see Jang and Sacht (2016) for an overview). The standard deviations  $\sigma_c$  and  $\sigma_\pi$  are given by 0.543 and 0.240, respectively, which show moderate exogenous shocks to consumption and inflation. With respect to the monetary policy parameters, the US Federal Reserve Bank follows an ambitious 'leaning-against-the-wind'-strategy ( $\phi_\pi = 1.914$ ), while it also reacts strongly to consumption movements ( $\phi_c = 0.709$ ). The Taylor rule exhibits a large degree of interest rate smoothing ( $\phi_r = 0.808$ ).

The peculiarity of bounded rationality can be measured by the parameter  $\beta$ , which is estimated to be 3.282. This suggests that US optimistic consumers believe in a deviation of the future value of consumption from its steady state by around plus ( $\beta/2 =$ )1.6 percent on average upwards. Due to the symmetry in the heuristics applied, pessimists assume a negative deviation from trend by around minus 1.6 percent on average downwards over the underlying time

interval. As mentioned before, the estimated degree of divergence denoted by  $\delta$  is insignificant. Furthermore, the result of the estimation for the speed of convergence reveals almost a corner solution with  $\psi_c = 0.951$ . According to the corresponding heuristic, equation (6) collapses into  $E_t^F c_{t+1} \approx c_{t-1}$ . The economic interpretation of the results is that the US consumers, who adopt this rule-of-thumb behavior, do not believe in a convergence of the future value in consumption towards its steady state. Instead, they judge the past realization of  $c_t$  with one lag as the most reasonable forecast of this variable. The label ‘fundamentalists’ seems to be misleading in this case as the expectation formation scheme of this group remains independent of the fundamental value itself but becomes purely backward-looking instead. This observation stays, of course, at odds with the rational expectation hypothesis, i.e. the cornerstone of standard macroeconomic modeling. As it is already discussed above, our result suggests that the model under the PTB scenario leads to an equal (truly) good fit to the data like the EFB one. This reinforces our view that a purely backward-looking expectation formation scheme will dominate. Therefore it is shown in PTB that  $\psi_c$  and  $\xi_c$  are both also estimated to be close to one (cf. equations (6) and (7) for details).

The results for the *Euro Area* are presented in Table 3 below. Again, in the RE case  $J$  amounts to the highest value across all specifications given by 230.49 except for the pure emotional block (PEB) scenario with  $J = 261.71$ . This observation leads to new insights of the results presented in Jang and Sacht (2016), who focus on the empirical performance of RE versus PEB only. First, the result confirms that both scenarios exhibit an equal fit to the data as already mentioned in the previous study. Second, this fit is worst judged by the value of  $p$  which is equal to zero in both cases. This is in fact a contradiction to Jang and Sacht (2016), who find consistently lower (higher) values of  $J$  ( $p$ ) for RE and PEB. The difference between both studies is heavily grounded on the fact that the authors consider a hybrid RE model which allows some scope for backward-looking terms in the IS and NKPC. Up to this point, in this follow-up paper we focus on the purely-forward looking NKM instead (cf. the discussion on that issue in our model section). We turn to the hybrid version of the NKM in the next Section. Here we discuss the need for leads and lags in the structural equations of the model in order to capture inertia in the corresponding time series.

The bounded rational expectations scenario, which exhibits the best fit of the corresponding model specification to the Euro Area data, is given by the pure technical block (PTB). Like for the US economy, the associated value of  $J$  is close to the other scenario of the cognitive aggregate block (CAB): 37.96 versus 38.15. See also the corresponding  $p$ -values given by 0.999 and 0.998. It has to be mentioned here that the CAB scenario includes all four different heuristics for consumers’ expectation formation, while the dynamics in the PTB case are driven by those for fundamentalists and chartists only. However, it leaps to the eye that the bounded rationality parameters  $\beta$  and  $\delta$  are estimated to be insignificant — which limits the practical use of the CAB scenario for policy

**Table 3:** Estimation results based on Euro Area data (RE & BR scenarios)

Label	RE	CAB	PEB	PTB	EFB	ECB
$\tau$	0.000 -	0.154 0.0 - 0.312	0.321 0.0 - 0.824	0.144 0.005 - 0.284	0.569 0.356 - 0.782	0.319 0.135 - 0.503
$\sigma_c$	1.288 1.050 - 1.527	0.370 0.118 - 0.622	0.471 0.0 - 1.155	0.413 0.206 - 0.619	0.839 0.716 - 0.962	0.474 0.131 - 0.817
$\kappa$	0.280 0.093 - 0.468	0.159 0.121 - 0.197	0.225 0.082 - 0.368	0.152 0.125 - 0.178	0.166 0.133 - 0.200	0.171 0.111 - 0.230
$\sigma_\pi$	1.087 0.793 - 1.380	0.354 0.172 - 0.536	0.326 0.160 - 0.492	0.360 0.213 - 0.507	0.401 0.214 - 0.587	0.382 0.207 - 0.558
$\phi_\pi$	1.000 -	1.612 1.075 - 2.150	1.571 1.0 - 2.151	1.593 1.056 - 2.129	1.262 1.0 - 1.609	1.360 1.0 - 1.816
$\phi_c$	1.804 0.730 - 2.878	0.309 0.026 - 0.593	0.336 0.030 - 0.642	0.325 0.039 - 0.611	0.571 0.345 - 0.797	0.476 0.196 - 0.756
$\phi_r$	0.721 0.637 - 0.805	0.420 0.222 - 0.618	0.381 0.124 - 0.638	0.426 0.229 - 0.623	0.602 0.498 - 0.706	0.505 0.331 - 0.679
$\sigma_r$	0.002 -	0.429 0.035 - 0.824	0.288 0.0 - 0.662	0.444 0.078 - 0.809	0.258 0.0 - 0.518	0.331 0.0 - 0.680
$\beta$	-	1.861 0.0 - 4.427	2.162 1.099 - 3.224	-	3.093 0.0 - 6.504	1.509 0.0 - 3.594
$\delta$	-	1.709 0.0 - 4.397	0.591 0.0 - 1.352	-	2.000 0.0 - 5.393	1.225 0.0 - 3.790
$\psi_c$	-	0.570 0.139 - 1.0	-	0.762 0.526 - 0.998	0.945 0.630 - 1.0	-
$\xi_c$	-	1.500 0.683 - 2.316	-	1.010 0.574 - 1.447	-	0.798 -0.132 - 1.728
$J$	230.49	38.15	261.71	37.96	42.11	46.85
$p$	0.000	0.998	0.000	0.999	0.993	0.971

*Note:* We use 78 moments (two years), based on the SMM approach. The 95% confidence intervals are given with a smaller size. The value of the objective function and the p-value are denoted by  $J$  and  $p$ , respectively. The degrees of freedom for  $\chi^2$  distribution amount to 66, 67, 68 (horse race) and 70 (RE, pure[ly forward-looking] BR). The 5% critical values for 66, 67, 68, 70 degrees of freedom are 85.96, 87.11, 88.25 and 90.53, respectively. No memory is assumed in the BR scenarios ( $\rho = 0$ ). The discount factor  $\nu$  is calibrated to 0.99. We set  $\varpi$  equal to 1800.

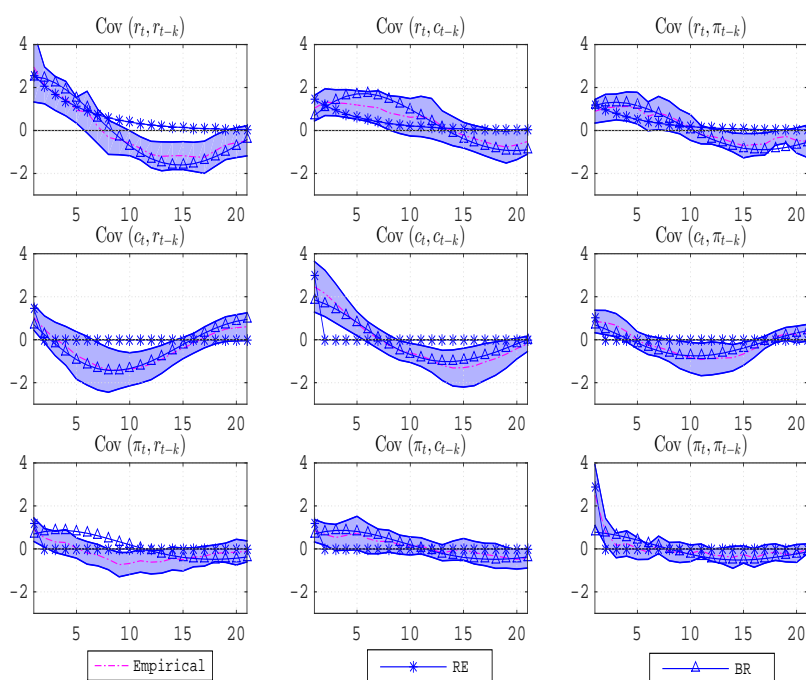
analysis. Hence, we conclude that the PTB serves as the most appropriate choice as one notices that here none of the parameters are insignificant. It can be clearly seen that in line with the results for the US economy, scenarios which consist on technical heuristics outperform the one with emotional ones being considered only (PEB) and, most importantly, the RE scenario. Once again it becomes apparent that lags of first and second order in  $c_t$  are non-negligible components in this type of business cycle model.

Now we turn to the interpretation of the parameter estimates for the PTB scenario. In particular, the parameter values for  $\tau = 0.144$  and  $\kappa = 0.152$  indicate lower pass-through effects on consumption and inflation compared to the US case. These values are therefore close to the ones under RE obtained in various studies (see, again, Jang and Sacht (2016) for an overview). The standard deviations of the shock to consumption and inflation are moderate with  $\sigma_c = 0.413$  and  $\sigma_\pi = 0.360$ , respectively. The nominal interest rate shock is estimated to be close to these values with  $\sigma_r = 0.444$ , while the Taylor rule exhibits a much lower degree of persistence with  $\phi_r = 0.426$  than the US case. With respect to the latter, the European Central Bank reacts less aggressively to changes in consumption ( $\phi_c = 0.325$ ) but quite strongly (like the US Federal Reserve Bank) to inflation fluctuations ( $\phi_\pi = 1.593$ ).

Overall, the estimation results for the bounded rationality parameters reveal a high degree of backward-looking expectation formation. The fundamentalists believe in a moderate convergence rate of the consumption's future value towards its steady state under consideration of  $\psi_c = 0.762$ . This stands in contradiction to the US case where we observe a purely backward-looking scheme. However, the chartists' heuristic for the expectation formation process turns out to be a corner solution with  $\xi_c = 1.010$ : this type of agents purely extrapolate the past realizations of consumption, i.e. the corresponding expression  $E^C c_{t+1} \approx 2c_{t-1} - c_{t-2}$  (cf. equation (7)) holds. Again, these observations call for a model specification in the absence of rational expectations. This becomes apparent as the fit of the RE scenario differs significantly from those obtained from the bounded rational ones, which is, again, the same statement one could make with respect to the US case. Hence, based on our results it is recommend to consider the BR model for policy analysis.

Finally, we simulate the trajectories of consumption, inflation and the nominal interest rate based on the estimates from the RE and BR scenarios. For the US data, we choose the estimation results of EFB for simulations. Figure 3 below shows the auto- and cross-covariances for US data which are compared to their empirical counterparts. A good fit is mimicked by the observation that the simulated auto- and cross-covariances are placed within the confidence bands of the empirical ones. The RE model is not successful at matching the moments, while the BR model (under EFB) could approximate the empirical moments well. For example, the RE model fails to generate persistence in the covariance profiles for consumption and inflation. On the contrary, the BR model has the ability to generate the persistent behavior for consumption and inflation based

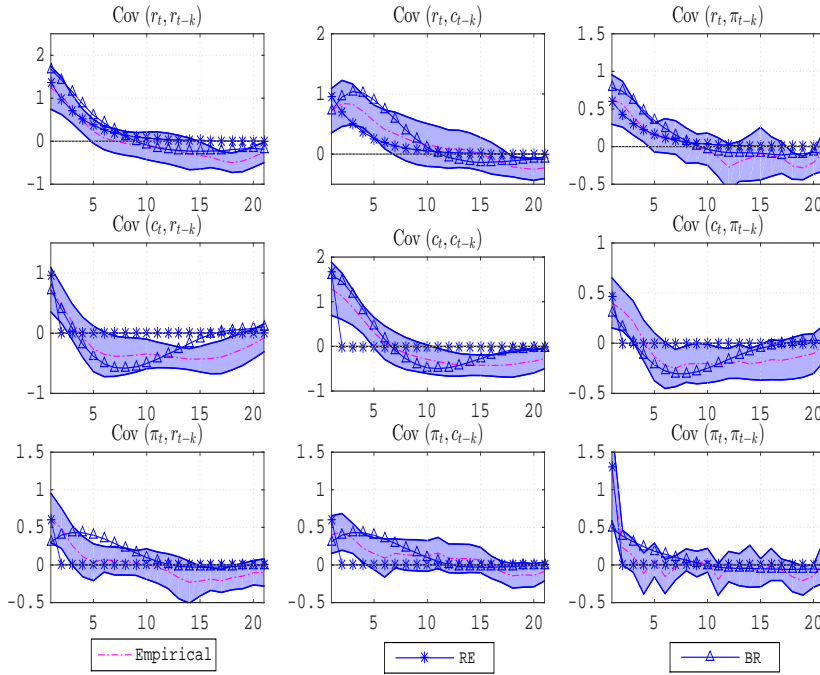
**Figure 3:** Model covariance in the profiles in US case



*Note:* The estimation results obtained under the emotional-fundamental block (EFB) are used to simulate the auto- and cross-covariance (COV) in the BR scenario.

on backward-looking expectations. This is confirmed by the simulated profiles of the auto- and cross-covariances based on the BR model. Next we consider the PTB scenario with respect to Euro Area data. The corresponding simulated auto- and cross-variances are plotted in Figure 4 below. The performances of the RE and BR models are both qualitatively similar to the results from US data. Hence, we can arrive at the same statement as above.

**Figure 4:** Model covariance profiles in Euro Area



*Note:* The estimation results obtained under the pure-technical block (PTB) are used to simulate the auto- and cross-covariance (COV) in the BR scenario.

To sum up, we support the rule-of-thumb behavior for the US and Euro Area data. Straight to the point, approximately pure backward-looking (instead of rational) expectation formation can be identified as an appropriate choice for modeling consumer confidence and, hence, forecast evaluation. The difference in both economic regions is given by the influence of emotional states on decision making in the US compared to the Euro Area, where expectation formation is more technical in nature. The remarkable goodness of fit of the BR scenarios (i.e., EFB and PTB) indicates that optimism and pessimism, as well as trend following and fundamental oriented forecasting behavior, play a major role in expectation formation.<sup>7</sup>

<sup>7</sup>By sorting agents in one of these groups, however, we cannot state that they act entirely emotional or professional (in term of forecasting techniques being applied by this group). In practice, confidence is influenced by many factors which might not be captured completely by the heuristics being considered in this paper. This calls for an empirical validation of different heuristics in future studies.

For both economic regions it is needless to say that the incorporation of backward-looking elements is required in order to describe macroeconomic dynamics well. Under consideration of forward-looking elements only, which are stemming from the rational expectation paradigm, those are not sufficient enough to establish a good fit to the data. This statement does not rule out the possibility for low values of the objective function where we allow for hybridity in the NKM under rational expectations. Hence, we introduce intrinsic persistence through the well-known standard concepts of habit formation in consumption and price indexation. The corresponding comparison of this model type to the ones under bounded rationality is done in the following Section.

## 5 Is There a Need for Hybridity in Decision Making?

We consider the hybrid version of the RE model as a comparison to the BR models. This comparison exercise between both models can be used to examine the similarity and differences between backward-looking terms in the two models, because these terms play a role for intrinsic persistence in the dynamics. More precisely, the baseline hybrid three-equations NKM reads as follows:

$$c_t = \frac{1}{1+\chi} \tilde{E}_t^{RE} c_{t+1} + \frac{\chi}{1+\chi} c_{t-1} - \tau(r_t - \tilde{E}_t^{RE} \pi_{t+1}) + \varepsilon_{c,t} \quad (17)$$

$$\pi_t = \frac{\nu}{1+\gamma\nu} \tilde{E}_t^{RE} \pi_{t+1} + \frac{\gamma}{1+\gamma\nu} \pi_{t-1} + \kappa c_t + \varepsilon_{\pi,t} \quad (18)$$

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)(\phi_\pi \pi_t + \phi_c c_t) + \varepsilon_{r,t} \quad (19)$$

$$c_t = y_t \quad (20)$$

where hybridity is introduced using the parameters for habit formation  $0 \leq \chi \leq 1$  and price indexation  $0 \leq \gamma \leq 1$ , respectively. All definitions of the remaining parameters and variables remain the same as before except for the lags in (17) and (18) now being considered explicitly. Note here that we consider the stylized version of the well-known Smets and Wouters (2003, 2005, 2007) model. This model specification addresses the so-called ‘(inflation) persistence problem’ discussed by Chari et al. (2000), who state that only shock-driven models without intrinsic persistence cannot account for inertia in the data. In the following we report the matching of the second theoretical moments to the empirical ones of these stylized hybrid version (cf. Franke et al. (2015)). We compare the goodness-of-fit to those models underlying the BR scenarios EFB and PTB only. This is done due to the fact that both specifications lead to best fit to the data in the US and Euro Area case, respectively, as reported in the previous Section.

The estimation results are shown in Table 4 below. The results highlight the importance of backward-looking behavior for the empirical application to both the US and Euro Area. This can be seen by the estimates for the habit formation and price indexation parameters in the RE cases which are both to be close (for  $\gamma$ ) or even at their boundary value of unity (for  $\chi$ ).

**Table 4:** Estimation results for the hybrid RE model versus the BR model

Label	US data		Euro Area data	
	Hybrid RE	EFB	Hybrid RE	PTB
$\chi$	1.000 -	-	1.000 -	-
$\tau$	0.032 0.015 - 0.048	0.371 0.222 - 0.520	0.079 0.022 - 0.136	0.144 0.005 - 0.284
$\sigma_c$	0.554 0.394 - 0.714	0.543 0.267 - 0.818	0.561 0.430 - 0.693	0.413 0.206 - 0.619
$\gamma$	0.914 0.803 - 1.0	-	0.765 0.630 - 0.900	-
$\kappa$	0.030 0.019 - 0.040	0.213 0.175 - 0.252	0.035 0.021 - 0.049	0.152 0.125 - 0.178
$\sigma_\pi$	0.293 0.153 - 0.434	0.240 0.018 - 0.461	0.275 0.159 - 0.390	0.360 0.213 - 0.507
$\phi_\pi$	1.573 1.000 - 2.228	1.914 1.080 - 2.747	1.288 1.0 - 1.918	1.593 1.056 - 2.129
$\phi_c$	0.785 0.253 - 1.317	0.709 0.011 - 1.407	0.497 0.124 - 0.870	0.325 0.039 - 0.611
$\phi_r$	0.831 0.766 - 0.895	0.808 0.660 - 0.956	0.604 0.479 - 0.729	0.426 0.229 - 0.623
$\sigma_r$	0.464 0.133 - 0.796	0.151 0.000 - 0.417	0.421 0.072 - 0.769	0.444 0.078 - 0.809
$\beta$	-	3.282 1.598 - 4.967	-	-
$\delta$	-	0.531 0.000 - 1.550	-	-
$\psi_c$	-	0.951 0.657 - 1.244	-	0.762 0.526 - 0.998
$\xi_c$	-	-	-	1.010 0.574 - 1.447
$J$	47.33	43.29	56.30	37.96
$p$	0.973	0.989	0.844	0.999

*Note:* We use 78 moments (two years), based on the SMM approach. The 95% confidence intervals are given with a smaller size. The value of the objective function and the p-value are denoted by  $J$  and  $p$ , respectively. For the hybrid RE, the degrees of freedom for  $\chi^2$  distribution amount to 68. The 5% critical value for 68 degrees of freedom is 88.25. No memory is assumed in the BR scenarios ( $\rho = 0$ ). The discount factor  $\nu$  is calibrated to 0.99. We set  $\varpi$  equal to 1800.



In direct comparison to the performance of the hybrid RE model to the one under BR, the latter exhibits equal (US case) or even better fit (Euro Area case) to the data based on the  $J$  statistics while the  $p$ -values turn out to be high. This remarkable observation is likely due to the fact that there are high weights being considered in the heuristics of fundamentalists and chartists with respect to past realizations of consumption. Note, once again, that the corresponding expressions become  $E_t^F c_{t+1} \approx c_{t-1}$  and  $E_t^C c_{t+1} \approx 2c_{t-1} - c_{t-2}$ . This suggests that the BR model specifications developed in this study can be regarded as an alternative explanation for the expectation formation process in terms of intrinsic persistence compared to the one stemming from the hybrid RE model. To go one step further, we state that based on our results, forward-looking components do not become crucial in describing consumption expenditure and macroeconomic dynamics in general. In addition, the BR heuristics of these types provide novel insights into backward-looking behavior in the absence of rational expectations compared to simple habit formation and price indexation. This holds due to the rigorous economic explanations for the BR heuristics in terms of the speed of convergence ( $\psi_c$ ) and the parameter for extrapolation/reversal belief ( $\xi_c$ ).

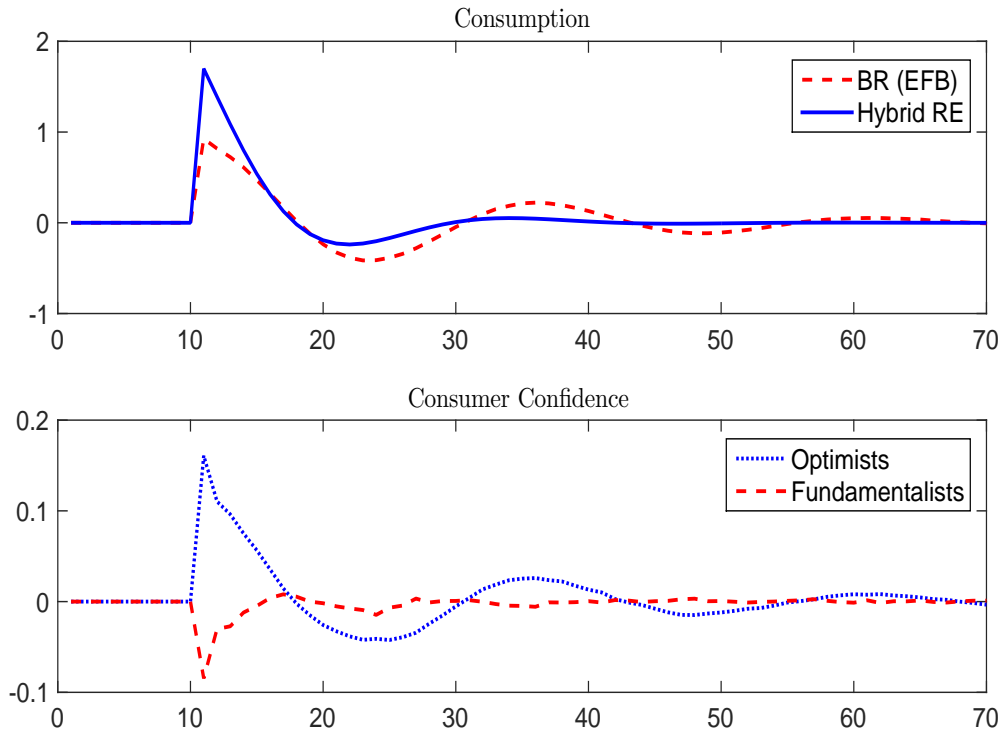
In the following, we compare the impulse response functions (IRF henceforth) in case of a demand shock across the hybrid version of the RE model and its BR counterparts for the US and Euro Area.<sup>8</sup> Based on our observations that all three scenarios exhibit almost the same good fit to the data, it will be fruitful to study the related outcome used for policy analysis. In order to compute the IRF, we display the deviation of the simulated time series from the same time series with an increase in  $\varepsilon_{c,t}$  by one unit being considered. The underlying simulations are both caused by the same series of random shocks. Our focus is on consumption (cf. equation (17) for the hybrid RE case together with the equilibrium condition (20)) and consumer confidence (defined by the corresponding heuristics). For the latter the IRF can be interpreted as follows. For the US economy, in the first step, according to the EFB scenario we consider two fractions of group for two different cases: optimists relative to pessimists *and* fundamentalists versus fundamentalists relative to (let's say) the emotional consumers (optimists *and* pessimists). Both specific fractions are computed in response to the shock. In the second step, we calculate the same relation in the absence of the shock. In the third and final step, the IRF is given by the deviation of the relations without the shock from the one where the shock occurs at time  $t = 10$ . The same computations are applied for the Euro Area, where we consider the change in the fraction of fundamentalists relative to chartists according to the PTB scenario. The IRF for consumer confidence can be seen as a *measure of dominance* for the heuristics being considered. Hence, in the

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<sup>8</sup>For completeness, the remaining IRF for a supply and a policy shock can be found in the Appendix A2. Since we are interested in the relationship between consumption and consumer confidence, in this paper we focus on the demand shock only. However, we would like to emphasize that the analysis of a supply shock is the most interesting one to be studied with respect to conducting optimal monetary policy (due to the output/inflation trade-off the central bank faces in this case).

US case, positive realizations above zero indicate a stronger increase in the fraction of optimists relative to the other two groups. The same holds with respect to the fraction of fundamentalists relative to the emotional consumers. For the Euro Area case, a value above zero indicates the dominance of the fundamentalists over the chartists. According to the group behavior, negative realizations below zero shows that the corresponding group is dominated by the other. Given this information we analyze the development of the IRF with respect to consumption.

**Figure 5:** Impulse response functions in case of a positive demand shock (US)



*Note:* The upper and lower panels display the impulse response functions with respect to consumption and consumer confidence, respectively. The latter shows the dominance of one group over the other group(s) of consumers. The deviation from steady state in percent are shown at the vertical axes. The periods in quarterly magnitudes are displayed at the horizontal axes. The shock hits the economy in period  $t=10$ . Both model specifications are calibrated according to the parameter estimates given in Table 4.

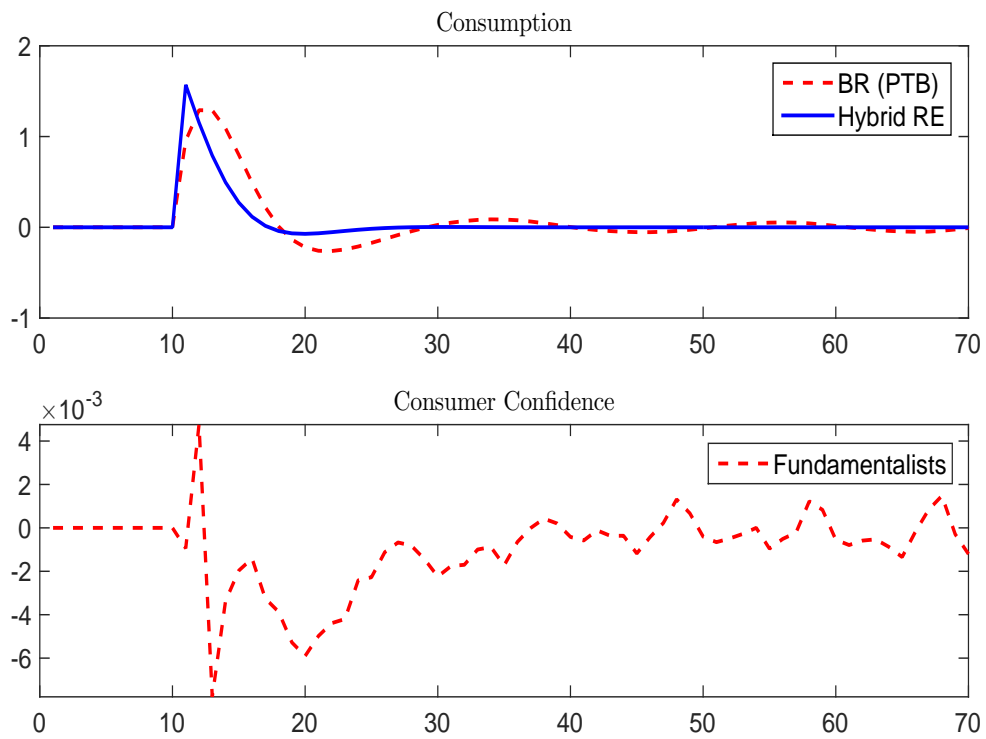
Figure 5 above represents the dynamics of consumption (top panel) and consumer confidence (lower panel) in the US case. The results support the prediction of the model when the economy is hit by a positive demand shock, i.e. an exogenous increase in consumption. The difference between the hybrid RE and BR scenario(s) lies in the fact that there are more wiggles and fluctuations in the system under BR than RE. This supports that under the assumption of bounded rationality the economy becomes more unstable. At the same time

the demand shock has a stronger impact on consumption in earlier periods in the hybrid RE case where the opposite is true for the BR case in later periods. The less pronounced effect on consumption on impact under EFB is caused by the dominance of optimists who expect a subjected mean value of plus  $\beta/2 = 1.621$  according to Table 4. The realized increase in consumption is less than the subjected mean value due to the existence of the pessimists who consider a negative value of  $\beta/2$  of the same amount. Note that the fundamentalists are clearly dominated on impact since this group simply expects the (unaltered) previous consumption level to be realized. Regarding the development of consumer confidence we observe the following. While the *realization* of consumption relies on the dominance of the optimistic group, the *volatility* in consumption depends on the dominance of the fundamentalists. The latter observations suggest that over time the dominance of one group over the other alternates: since agents switch from a technical to a more emotional grounded expectation formation and vice versa, the impact of the shocks prevails. This is indeed an indication for a high degree of autocorrelation in consumer confidence as we discussed in Section 2. The effects is dampened in the RE case due to perfect foresight. Our analysis also highlights the importance of relevant policies during the transition period because a central bank faces different dynamic patterns in consumption based on the degree of rationality.

Figure 6 below shows the outcome of the same experiment for the Euro Area. There is a rapid change in the dominance of the fundamentalists over the chartists for the first five periods. This leads to a strong increase in consumption like in the hybrid RE case. Both IRF for consumption indicate almost identical dynamic patterns which can be explained by the fact that in both model specifications there is a high weight on backward-looking expectation formation. However, consumer confidence becomes more volatile under BR, which can be explained by the dominance of the chartists. Indeed, this group extrapolates into the future under consideration of the past realizations of consumption up to the second lag. This dominance leads to the highest peak (around period twelve) and lowest trough (around period twenty-one) which coincide with the fact that the fundamentalists are highly dominated in the corresponding periods (note that in these cases the IRF are below zero). The increase in the relative fraction of the fundamentalists after period twenty-one contributes to a decrease in volatility over time. However, in the absence of perfect foresight, volatility in consumption prevails in the BR case while the opposite holds for the hybrid RE model. Again, the high degree of autocorrelation in consumer confidence then contributes to consumption volatility as mentioned in Section 2.

Our results address the challenges that policy makers face, especially when stimulating the economy via fiscal and/or monetary policy in the presence of animal spirits. For example, only few studies have investigated (optimal) monetary and fiscal policy under bounded rationality (cf. Caprioli (2015), De Grauwe and Macchiarelli (2015), Hollmayr and Matthes (2015) as well as Lengnick and Wohltmann (2016) among others). Cornea-Madeira et al. (2017) state that due to the existence of multiple equilibria in a complex system under bounded

**Figure 6:** Impulse response functions in case of a positive demand shock (Euro Area)



*Note:* See Figure 5.

rationality, valid empirical evidence for behavioral heterogeneity is questioning the formulation of optimal policy design under the rational expectation paradigm. In fact, the results presented in this paper are in line with the ones by Cornea-Madeira et al. (2017). The latter show that heterogeneity varies over time where they conclude that inflation dynamics can be dominated by either forward-looking or backward-looking behavior. Indeed, according to the  $J$  values in Table 4, it is apparent that a hybrid model with forward-looking elements cannot be ruled out — at least for the US economy. This requires, however, the existence of a lag in the structural equations as we also know from the previous section based on the  $J$  statistics. Our results indicate that further research on optimal policy under incorporation of the rule-of-thumb behavior into the economic modeling is needed. In particular, our parameter estimates obtained in this Section stand out as a point of departure for this kind of experiments.

## 6 Conclusion

In this paper we empirically examine the baseline New-Keynesian model with heterogeneous agents who may adopt various heuristics used to forecast future movements in consumption. In this framework, these specific forecast heuristics are applied when forming consumption expectations due to discrete choice. Based on their favorable expectation scheme, agents sort themselves into the four groups of optimists, pessimists, fundamentalists or chartists, who adopt backward-looking expectation formation via rule-of-thumb behavior. The corresponding competing non-linear specifications of the model under bounded rationality are estimated via the Simulated Method of Moments approach.

The “wilderness of bounded rationality problem” (Hommes (2011)) makes the formulation of bounded rationality models a non-trivial one. This statement relies on the fact that agents’ ability and willingness of possessing information when making decisions lead to challenges when it comes to the modeling of the underlying neurological processes. Regarding expectation formation and forecasting it is not only important to investigate how corresponding heuristics are conducted but also whether those are empirically relevant. While some kind of wilderness manifests itself in a high degree of freedom to develop bounded rationality models, this paper focuses on the modeling and validation of consumer confidence along an emotional and technical dimension. Our empirical results show that expectation formation in the US is heavily grounded on agents’ emotional state (with respect to optimism and pessimism), while for the Euro Area it is most likely technical in nature (with respect to fundamentalists and chartists). We argue that this study contributes to the estimation of bounded rationality models used in macroeconomic research and policy analysis. As the most interesting result, we show that the model specifications under bounded rationality outperform significantly the one under rational expectations with only forward-looking components. Furthermore, the corresponding consumers’ heuristics exhibit a high degree of backward-looking behavior which results in a collapse of these expressions into corner solutions with the associated pa-

rameters being estimated close to unity. This becomes most apparent as the superiority of bounded rationality models prevails when it is compared to the hybrid version of the baseline NKM with leads and lags. This observation questions the need for a hybrid specification of DSGE models under a forward-looking expectation formation scheme.

Our observation reveals that for policy analysis, a model under bounded rationality serves as a reliable alternative approach. The question then arises whether our results obtained through our horse race exercise are sufficient enough to examine different heuristics for bounded rationality. Similarly, such an experimental setting should account for agents' confidence in investment decisions. Therefore, additional moment conditions besides the auto- and cross-covariance profiles (like the raggedness of the time series; see Franke (forthcoming) for more details) can also be considered when estimating DSGE models via the Simulated Method of Moment approach. Further future attempts should focus on the relationship between bounded rational heuristics and the stability of the model. Research related to this topic is still at an early stage of development. As a point of departure the methodology applied in Chiarella et al. (2009) is a promising one. The authors show that models from financial economics with fundamentalists and chartists is able to produce an outcome of a locally stable fundamental equilibrium while it also accounts for the stylized facts of financial markets at the same time. In general, the stability analysis in any kind of model where highly non-linear bounded rational heuristics are considered is crucial for conducting reliable policy measures — especially in macroeconomic business cycle models with the focus on optimal monetary and fiscal policy. We leave all these important topics to future research.

## Appendix

### A1: The Simulated Method of Moments (SMM) Approach

In this paper we seek to match the model-generated covariances across all scenarios of consumption, inflation and the nominal interest rate (all in gap notation) with their empirical counterparts. Statistical inference on the market behavior is based on those model parameter values. The parameter estimates are considered as the result of the minimization of the distance between the model generated and empirical second moments in SMM. The moments include the variances of the model's variables (i.e. their absolute volatility), while pro- and counter-cyclical movements in the different aggregates are captured by (not only the first-order but) an entire profile of the auto- and cross-covariances. Hence, we can make use of all (unconditional) co-movement statistics characterized by the estimation. See Franke et al. (2015) for a detailed introduction to this method and Jang and Sacht (2016) for an application to a DSGE model under bounded rationality.

More generally, the moment conditions account for the distributional properties of empirical data  $X_t$  with  $t = 1, \dots, T$ ; where  $T$  denotes the sample size. The sample covariance matrix at lag  $k$  is defined by:

$$m_{\mathbb{T}}(k) = \frac{1}{T} \sum_{t=1}^{T-k} (X_t - \bar{X})(X_{t+k} - \bar{X})' \quad (21)$$

where  $\bar{X} = (1/T) \sum_{t=1}^T X_t$  is the vector of the sample mean. The sample average of discrepancy between the model-generated and empirical moments is denoted as

$$g(\theta; X_t) \equiv \frac{1}{T} \sum_{t=1}^T (m_{\mathbb{T}}^* - m_{\mathbb{T}}) \quad (22)$$

where  $\theta$  is a  $l \times 1$  vector of unknown structural parameters.  $m_{\mathbb{T}}^*$  and  $m_{\mathbb{T}}$  are the *empirical* and the *model generated* moment functions, respectively (cf. equation (21)).

The main goal of this study is to compare the performance of several behavioral macroeconomic specifications (as described in the previous section) based on observations' auto- and cross-covariances at a (fixed) lag  $k$  with  $k = 0, \dots, n$ . After selecting an appropriate number of  $j$  variables for the lag length, we compute the corresponding  $p$ -dimensional vector of (empirical and simulated) moment conditions:

$$p = p(k, j) = (j \cdot k - 1) \cdot j. \quad (23)$$

We avoid the double counting at the zero lags in the cross relationships by considering the term  $(j \cdot k - 1)$ . Note here that the Delta method is used to construct a confidence interval for the auto- and cross-covariance moments. The lag length is then given by  $n = k^{max} = 8$  since repeating patterns in the

time series do not exhibit additional information, while the model has three variables ( $j = 3$ ). Therefore, we consider  $p = 78$  moments as an appropriate choice according to equation (23) and the underlying model structure (see also Jang and Sacht (2016)).

Matching these moment conditions has an impact on both parameters and empirical aspects of interest, and we can estimate the model parameters by minimizing the following quadratic objective function:

$$J(\theta) = \min_{\theta} g(\theta; X_t)' \widehat{W} g(\theta; X_t) \quad (24)$$

where more importance is attached to particular moment conditions according to the weighting matrix  $\widehat{W}$ . The kernel estimator has the following general form with the covariance matrix of the appropriately standardized moment conditions given by

$$\widehat{\Gamma}_T(h) = \frac{1}{T} \sum_{t=h+1}^T (m_T - \bar{m})(m_T - \bar{m})' \quad (25)$$

where  $\bar{m}$  once again denotes the sample mean. The popular choice of  $h \sim T^{1/3}$  is used to find an appropriate lag length, that is,  $h = 5$  for estimating the covariance matrix in the Euro Area (i.e. the Hansen-White covariance estimator):

$$\widehat{\Omega} = \widehat{\Gamma}_T(0) + \sum_{h=1}^5 \left( \widehat{\Gamma}_T(h) + \widehat{\Gamma}_T'(h) \right). \quad (26)$$

The weighting matrix  $\widehat{W}$  is computed from the inverse of the estimated covariance matrix  $\widehat{\Omega}$ . However, the estimated covariance exhibits a singularity at a point where high correlations between the moment conditions occur. This leads to an increase in the correlation between the moment conditions and the weighting matrix. The singularity problem of the covariance matrix will be a big issue for small sample data (Altonji and Segal (1996)). To circumvent the econometric issues, we use the diagonal matrix entries as the weighting scheme, while the off-diagonal components of the matrix  $\widehat{W} = \widehat{\Omega}^{-1}$  are ignored.

Now we examine the properties of the sample distribution for the parameter estimation. In particular, under certain regularity conditions, we arrive at the following asymptotic distribution of the model parameters:

$$\sqrt{T}(\widehat{\theta}_T - \theta_0) \sim N(0, \Lambda) \quad (27)$$

where  $\Lambda = [(DWD')^{-1}]D'W\Omega WD[(DWD')^{-1}]'$  holds.  $D$  is the gradient vector of moment functions evaluated around the point estimates. This can be written as:

$$\widehat{D} = \left. \frac{\partial m(\theta; X_T)}{\partial \theta} \right|_{\theta = \widehat{\theta}_T}. \quad (28)$$



Under RE, we obtain the simple analytic moment conditions of the model as described above. However, the analytic expressions of the BR model are not available, because the non-linear structure of the expectation formation process itself puts constraints on the model dynamics. To circumvent this problem, we simulate the BR model and estimate the behavioral parameters. In particular, SMM is suited to a situation where the model is easily simulated by replacing theoretical moments. Then the model-generated moments in equation (24) are replaced by their simulated counterparts:

$$m_T = \frac{1}{S} \sum_{s=1}^S \tilde{m}_T. \quad (29)$$

In equation (29), gives us the approximation of theoretical moments ( $m_t$ ) with the simulated data of  $\tilde{m}_t$ . The simulation size is denoted by  $S$ . Under certain regularity conditions, the SMM estimator is asymptotically normal (Duffie and Singleton (1993), Lee and Ingram (1991)):

$$\sqrt{T}(\hat{\theta}_{SMM} - \theta_0) \sim N(0, \Lambda_{SMM}), \quad (30)$$

where  $\Lambda_{SMM} = [(B'WB)^{-1}]B'W(1 + 1/S)\Omega WB[(B'WB)^{-1}]'$  holds, i.e. the covariance matrix of the SMM estimates. A gradient vector of the moment function is defined as  $B \equiv E\left[\frac{\partial m_t}{\partial \theta} \Big|_{\theta=\hat{\theta}}\right]$ .

However, the model estimation contains simulation errors, in which we could not easily arrive an accurate approximation of the non-linear expectation formation processes in the BR model. Alternatively, we compute the standard errors by using the following steps:

1. The BR model is estimated using a simulation size of  $S = 10$ .
2. The estimation is iterated over 100 times while different random seeds are used to obtain point estimates of the model parameters for each iteration.
3. We take 100 different estimates to compute the mean and standard error of the parameter estimates.

Indeed, the above iterative method is regarded as being equivalent to a single estimation of the model based on a simulation size of 1,000. Note here that the iteration approach can take the benefit of low simulation errors.

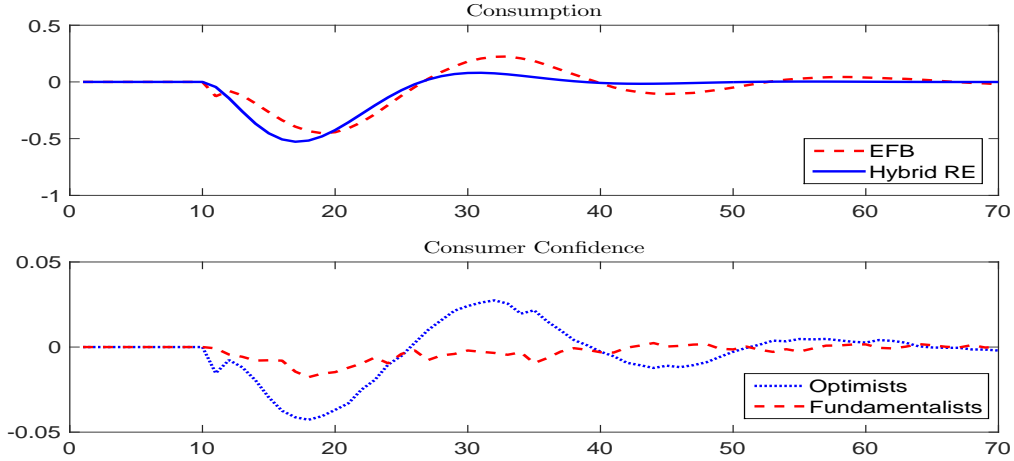
Finally, we use the  $J$  test to evaluate compatibility of the moment conditions:

$$\bar{J} \equiv T \cdot J(\hat{\theta}) \stackrel{d}{\rightarrow} \chi_{p-l}^2, \quad (31)$$

where  $l$  denotes the number of parameters to be estimated. In general, the  $J$ -statistic is asymptotically  $\chi^2$ -distributed with  $(p - l)$  degrees of freedom. In this study, the lag length for the covariance is set to two years. Hence, the number of moment conditions exceeds the model parameters and we consider this particular case as over-identification.

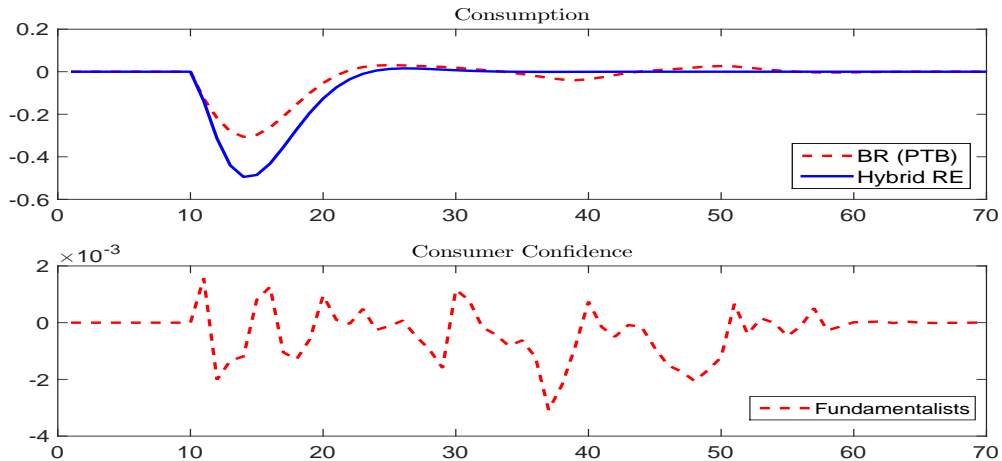
## A2: Impulse Response Functions for a Supply and a Monetary Policy Shock

**Figure 7:** Impulse response functions in case of a positive supply shock (US)



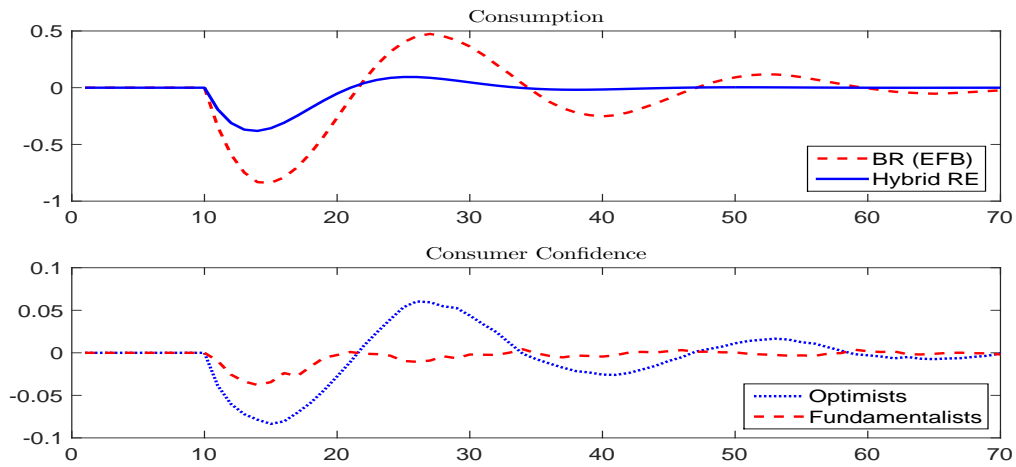
*Note:* The upper and lower panel displays the impulse response functions with respect to consumption and consumer confidence, respectively. The latter shows the dominance of one group over the other group(s) of consumers. The deviation from steady state in percent are shown at the vertical axes. The periods in quarterly magnitudes are displayed at the horizontal axes. The shock hits the economy in period  $t = 10$ . Both model specifications are calibrated according to the parameter estimates given in Table 4.

**Figure 8:** Impulse response functions in case of a positive supply shock (Euro Area)



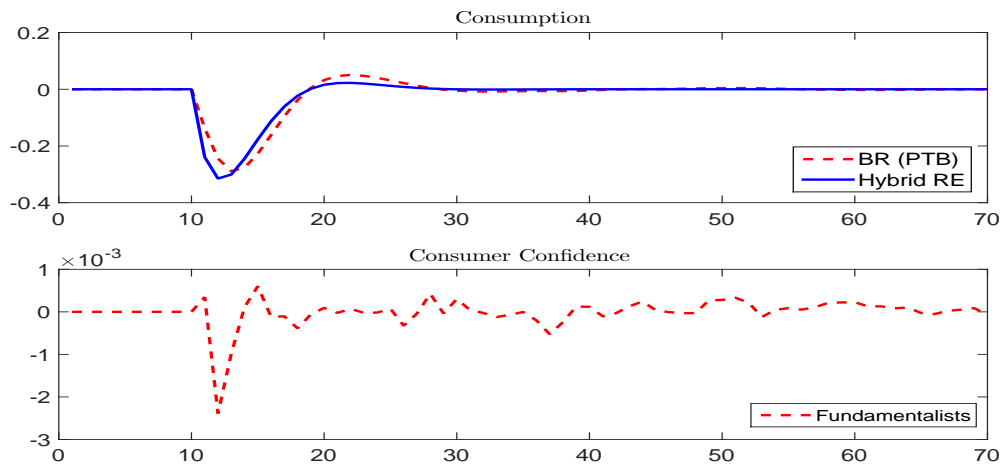
*Note:* See Figure 7.

**Figure 9:** Impulse response functions in case of a positive monetary policy shock (US)



*Note:* See Figure 7.

**Figure 10:** Impulse response functions in case of a positive monetary policy shock (Euro Area)



*Note:* See Figure 7.

## References

- Akerlof, G.A. and Shiller, R. J. (2009): ‘Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism’, University Press Group Ltd.
- Altonji, J. and Segal, L. (1996): ‘Small-sample bias in GMM estimation of covariance structures’, *Journal of Business and Economic Statistics*, Vol. 14, pp. 353–366.
- Anderson, G.S. (2008): ‘Solving linear rational expectations models: a horse race’, *Computational Economics*, Vol. 31(2), pp. 95–113.
- Asada, T. Chiarella, C. Flaschel, P. and Proaño, C.R. (2007): ‘Keynesian AS-AS quo vadis’, Working Paper No. 151, School of Economics and Finance, University of Technology Sydney.
- Barsky, R. B. and Sims, E. R. (2012): ‘Information, animal spirits, and the meaning of innovations in consumer confidence’, *American Economic Review*, 102(4), pp. 1343–1377.
- Blanchard, O. J. (1993): ‘Consumption and the recession of 1990–1991’, *American Economic Review*, 83(2), pp. 270–74.
- Bofinger, P. Debes, S. Gareis, J. and Mayer, E. (2013): ‘Monetary policy transmission in a model with animal spirits and house price boom and busts’, *Journal of Economic Dynamics & Control*, 37, pp. 2862–2831.
- Brock, W. and Hommes, C. (1997): ‘Rational routes to randomness’, *Econometrica*, 65, pp. 1059–1095.
- Caprioli, F. (2015): ‘Optimal fiscal policy under learning’, *Journal of Economic Dynamics and Control*, Vol. 58, pp. 101–124.
- Chari, V.V., Kehoe, P.J. and McGrattan, E.R. (2002): ‘Can sticky price models generate volatile and persistent real exchange rates?’, *Review of Economic Studies*, 69(3), pp. 533–563.
- Chiarella, C. Dieci, R. and He X.-Z. (2009): ‘Heterogeneity, Market Mechanisms, and Asset Price Dynamics’, in: Hens, T., Schenk-Hoppé, K.R. (Eds.): *Handbook of Financial Markets: Dynamics and Evolution*, North-Holland, pp. 277–344.
- Chiarella, C. Dieci, R. and He X.-Z. (2010): ‘Time-varying beta: a boundedly rational equilibrium approach’, Research Paper No. 275, Qualitative Finance Research Center, University of Technology Sydney.
- Chiarella, C. He, X.-Z. Huang, W. and Zheng, H. (2011): ‘Estimating behavioural heterogeneity under regime switching’, Research Paper No. 290, Qualitative Finance Research Center, University of Technology Sydney.
- Christiano, L.J., Eichenbaum, M. and Evans, C.L. (2005): ‘Nominal rigidities and the dynamic effects of a shock to monetary policy’, *Journal of Political Economy*, 113(1), pp. 1–45.
- Cornea-Madeira, A., Hommes, C. and Massaro, D. (2017): ‘Behavioral heterogeneity in U.S. inflation dynamics’, *Journal of Business and Economic Statistics*, forthcoming.
- De Grauwe, P. (2011): ‘Animal spirits and monetary policy’, *Economic Theory*, 47(2–3), pp. 423–457.

- De Grauwe, P. and Macchiarelli, C. (2015): ‘Animal spirits and credit cycles’, *Journal of Economic Dynamics and Control*, 59, pp. 95–117.
- Duffie, D. and Singleton, K. (1993): ‘Simulated moments estimation of Markov models of asset prices’, *Econometrica*, 61(4), pp. 929–952.
- Fagan, G., Henry, J. and Mestre, R. (2001): ‘An area wide model (AWM) for the Euro Area’, ECB working paper No. 42.
- Franke, R., Jang, T.-S. and Sacht, S. (2015): ‘Moment matching versus Bayesian estimation: backward-looking behaviour in the New-Keynesian baseline model’, *The North American Journal of Economics and Finance*, 31, pp. 126–154.
- Franke, R. (forthcoming): ‘Competitive moment matching of a New-Keynesian and an Old-Keynesian model’, *Journal of Economic Interaction and Co-ordination*, forthcoming.
- Galí, J. (2015): ‘Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications’, Second Edition, Princeton University Press.
- Gaunersdorfer, A. and Hommes, C. (2007): ‘A nonlinear structural model for volatility clustering’, in: Teysnière, G., Kirman, A. (Eds.): *Long Memory in Economics*, Springer, pp. 265–288.
- Gaunersdorfer, A., Hommes, C. and Wagener, F. (2008): ‘Bifurcation routes to volatility clustering under evolutionary learning’, *Journal of Economic Behavior & Organization*, 67(1), pp. 27–47.
- Golinelli, R. and Parigi, G. (2004): ‘Consumer sentiment and economic activity: a cross country comparison’, *Journal of Business Cycle Measurement and Analysis*, 2004/2, pp. 147–170.
- Gomes, O. (2010): ‘Consumer confidence, endogenous growth and endogenous cycles’, *Journal of Economic Studies*, 37(4), pp. 377–404.
- Hollmayr, J. and Matthes, C. (2015): ‘Learning about fiscal policy and the effects of policy uncertainty’, *Journal of Economic Dynamics and Control*, Vol. 59, pp. 142–162.
- Hommes, C. (2001): ‘Financial markets as nonlinear adaptive evolutionary systems’, *Quantitative Finance*, 1, pp. 149–167.
- Hommes, C. (2011): ‘The heterogeneous expectations hypothesis: some evidence from the lab’, *Journal of Economic Dynamics and Control*, 35, pp. 1–24.
- Keynes, J. M. (1936): ‘The General Theory of Employment, Interest and Money’, Cambridge University Press.
- Jang, T.-S. and Sacht, S. (2016): ‘Animal spirits and the business cycle: empirical evidence from moment matching’, *Metroeconomica*, 67(1), pp. 76–113.
- Lee, B. S. and Ingram, B. (1991): ‘Simulation estimation of time-series models’, *Journal of Econometrics*, 47(2-3), pp. 197–205.
- Lengnick, M. and Wohltmann, H.-W. (2016): ‘Optimal monetary policy in a new Keynesian model with animal spirits and financial markets’, *Journal of Economic Dynamics and Control*, 64(C), pp. 148–165.
- Liu, C. and Minford, P. (2014): ‘Comparing behavioural and rational expectations for the US post-war economy’, *Economic Modelling*, 43, pp. 407–415.

- Ludvigson, S. C. (2004): ‘Consumer confidence and consumer spending’, *Journal of Economic Perspectives*, 18(2), pp. 29–50.
- Milani, F. (2014): ‘Sentiment and the U.S. business cycle’, Working Paper No. 141504, Department of Economics, University of California-Irvine.
- OECD (2016a): ‘Household spending (indicator)’,  
doi: 10.1787/b5f46047-en, accessed on 23 July 2016. URL: <https://data.oecd.org/hha/household-spending.htm>
- OECD (2016b): ‘Consumer confidence index (CCI) (indicator)’,  
doi: 10.1787/46434d78-en, accessed on 23 July 2016. URL: <https://data.oecd.org/leadind/consumer-confidence-index-cci.htm>
- OECD (2016c): ‘Quarterly GDP (indicator)’,  
doi: 10.1787/b86d1fc8-en, accessed on 23 July 2016. URL: <https://data.oecd.org/gdp/quarterly-gdp.htm>
- Sacht, S. (2015): ‘On the efficiency of labor market reforms: how to solve the Spanish puzzle?’, *Economics: The Open-Access, Open-Assessment E-Journal*, 9(2015-44), pp. 1–28. URL: <http://dx.doi.org/10.5018/economics-ejournal.ja.2015-44>
- Smets, F. and Wouters, R. (2003): ‘An estimated dynamic stochastic general equilibrium model of the euro area’, *Journal of the European Economic Association*, 1, pp. 1123–1175.
- Smets, F. and Wouters, R. (2005): ‘Comparing shocks and frictions in US and euro area business cycles: a Bayesian DSGE approach’, *Journal of Applied Econometrics*, 20(2), pp. 161–183.
- Smets, F. and Wouters, R. (2007): ‘Shocks and frictions in US business cycles: a Bayesian DSGE approach’, *American Economic Review*, 97, pp. 586–606.
- van Aarle, B. and Kappler, M. (2012): ‘Economic sentiment shocks and fluctuations in economic activity in the euro area and the USA’, *Intereconomics*, 47(1), pp. 44–51.