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Animal Spirits and the Business Cycle: Empirical Evidence from Moment Matching

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Abstract

In this paper we empirically examine a hybrid New-Keynesian model with heterogeneous bounded rational agents who may adopt an optimistic or pessimistic attitude - so called animal spirits - towards future movements of the output and inflation gap. The model is estimated via the simulated method of moments using Euro Area data from 1975Q1 to 2009Q4. In addition, we compare its empirical performance to the standard model with rational expectations. Our empirical results show that the model-generated auto- and cross-covariances of the output gap, the inflation gap and the nominal interest gap can provide a good approximation of the empirical second moments. The result is mainly driven by a high degree of persistence in the output and inflation gap due to the impact of animal spirits on economic activity. Furthermore, over the whole time interval the agents had expected moderate deviations of the future output gap from its steady state value.

Keywords: Animal Spirits; Bounded Rationality; New-Keynesian Model; Simulated Method of Moments.

JEL classification: C53, D83, E12, E32.

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1 Introduction

Rational expectations are a flexible and common way of describing market behavior in dynamic stochastic general equilibrium (DSGE) models. Since the DSGE approach disposes a convenient analytic tractability under the assumption of rational expectations, the microfoundations of macroeconomic dynamics serve as an efficient toolbox for analyzing monetary and fiscal policy strategies. As Selten (2001 p. 2) states, however, "modern mainstream economic theory is largely based on an unrealistic picture of human decision theory". Indeed, experimental research on human behavior supports information processing with a limited cognitive ability instead of suggesting perfect information (see Hommes (2011) among others).

Keynes (1936) already attributed significant irrationality to the human nature and discussed the impacts of waves of optimism and pessimism - so called *animal spirits* - on economic activity.[1] In this paper, we show empirically that a behavioral approach with respect to the microfoundations can be used to help identifying the cognitive ability of economic agents and introducing a substantial degree of inertia in DSGE models. According to De Grauwe (2011), if agents are known to be either optimists or pessimists, then their ability (or better: limitation) to form their expectations affects economic activities, i.e. movements in employment, the output gap and the inflation rate, more appropriately than in standard rational expectations models. It follows that the assumption of animal spirits results in a *non-linear* behavior of bounded rational agents under consideration of discrete choice theory. Hence, optimistic or pessimistic expectations are considered as a kind of emergent behavior based on a stochastic switching rule.[2]

Indeed, in his paper, De Grauwe (2011) replaces the forward-looking elements in the baseline hybrid three-equations New-Keynesian Model (NKM) – which stem from the assumption of rational expectations – by a regime-switching mechanism based on animal spirits. While inertia in the dynamics of the output gap and the inflation rate (gap) is observed empirically, it is well known that the forward-looking NKM under rational expectations is not able to reproduce the corresponding IRFs without exogenous persistence induced by autocorrelated shock processes (cf. Chari et al. (2000) and Christiano et al. (2005)). As a possible alternative to rational expectations, the specification of a bounded rational expectation formation process can in fact account for intrinsic persistence in the output gap and the inflation rate – even if only non-autocorrelated exogenous shocks are considered. To the best of our knowledge, however, there is very little research into an empirical evaluation of a bounded rationality DSGE model of this type.

In this paper, we compare the empirical performance of a bounded rational-

[1] According to Akerlof and Shiller (2009), emotional states are reflected in economic behavior - see also Franke (2012) for his extensive discussion about market behavior and alternative ways to describe expectation formation processes in macroeconomic models.

[2] This procedure was first introduced by Brock and Hommes (1997) in their seminal paper on equilibrium models with adaptive learning (see also e.g. Westerhoff (2008) as well as Lengnick and Wohltmann (2013) among others).

ity model to a standard rational expectations model. In particular, similarities and dissimilarities between two polar cases of expectation formation processes will be examined: while the underlying model structure is identical to a baseline hybrid three-equations NKM, the two models differ in terms of the expectation formation process, namely, rational expectations and endogenously-formed expectations using the behavioral specification by De Grauwe (2011). In other words, we study his behavioral economic framework and provide an empirical investigation of bounded rationality on economic dynamics in the Euro Area from 1975Q1 to 2009Q4. In general, the behavioral modification of the NKM serves as an appropriate starting point for this kind of investigation, but it is inherently difficult to conduct the empirical analysis in the presence of non-linear mapping between the model and reduced form parameters. Hence, we aim to evaluate the potentially non-linear behavior of expectation formation processes via the *Simulated Method of Moments* (SMM) in the context of statistical inference.

Our main findings can be summarized as follows. First, our empirical results show that the model-generated auto- and cross-covariances of the output gap, the inflation gap and the nominal interest gap can provide a good approximation of the empirical moments. Here, the second moments represent specific aspects of the data generating process and, in particular, those which mimic one of the most important stylized facts of the economy. Second, the agents had expected moderate deviations of the future output gap from its steady state value over the whole time interval. These deviations show a strong correlation for contemporaneous changes based on booms and busts in economic activity. Third, one of the main results suggests strong evidence for a backward-looking expectation formation process, i.e. the previous realization of the inflation gap is strongly considered. This kind of expectation formation process has been discussed extensively by experimental economists (see Roos and Schmidt (2012) for an overview). Furthermore, the results indicate that the parameter estimates for the price indexation in both model specifications are close to unity. This result stands in contrast to other (empirical) studies in the New-Keynesian literature, which discuss the evidence for a purely forward-looking specification. Finally, we offer reliable point estimates that can be used for calibration exercises in the future work, e.g. studying monetary and fiscal policy analysis in a DSGE model without making the assumption of rational expectations.

Indeed, a plethora of studies have been done on alternative forms of information processing mechanisms in macroeconomics; see e.g. the literature on learning (Evans and Honkaphohja (2001)), rational inattention (Sims (2003)), sticky information (Mankiw and Reis (2002)) or bounded rationality in general (Sargent (1994) and Kahneman (2003)). Camerer (1998) also offers an informative overview of the discussion on these topics in economics. In addition, the results of previous studies include a discussion on the estimation of bounded rationality (mostly partial equilibrium) models. Therefore, our approach can be seen as related to the work of Brock and Hommes (1997), Milani (2007), Cornea et al. (2013) and Boswijk et al. (2007).

Brock and Hommes (1997) show that disequilibrium arises in a cobweb-type model when agents form rational and naive expectations – as being considered in De Grauwe (2011), and, of course, this paper. Milani (2007) estimates a linear hybrid NKM similar to the one in this paper with Bayesian techniques. As a feature of this model, the expectation formation process is described by constant-gain learning. According to the results of model comparison exercises, he shows that the model with constant-gain learning outperforms its rational expectation counterpart, where in the former case the learning process can generate a substantial degree of inertia in the model dynamics. However, the bounded rationality model studied in this paper is based on a non-linear switching rule from discrete choice theory. It is well known that Bayesian techniques can not be easily used to evaluate the non-linearity in this kind of models.

In another study, Cornea et al. (2013) estimate the NKPC for the US economy with an endogenous variation in the fractions of forward-looking fundamentalists and backward-looking native price-setters. The authors show that due to their theoretical modifications, their specification of the NKPC fits the data well. They conclude that this result is mainly driven by a regime switching mechanism, which is similar to the one presented in this paper. In general, the non-linear least squares approach used in Cornea et al. (2013) is in general based on the stringent assumptions about the underlying shock process (e.g. homoscedastic and non-autocorrelated shocks), while the effectiveness of this procedure might be limited to the case where the model is parsimonious. However, the results in their paper are not affected by this shortcoming, since they just consider a partial equilibrium model (for inflation rate dynamics), where the amount of parameters to be estimated is indeed manageable. In the same vain, Boswijk et al. (2007) estimate a dynamic single-equation asset pricing model with two types of agents. Behavioral heterogeneity in their paper is described by a high degree of switching between fundamentalists and chartists. However, the major difference in our approach is that making inferences about the group behavior is based on a system-of-equations estimation approach. Therefore, the current study aims to show that the SMM approach can be applied in order to improve the empirical shortcomings on non-linear behavioral dynamics within a highly parameterized model. To the best of our knowledge, such kind of investigation with respect to a non-linear DSGE model with *regime-switching* has not been undertaken in the literature so far.

The remainder of the paper is structured as follows. In section 2 we present the baseline NKM known from the literature and discuss the standard model with rational expectations and its variant under consideration of animal spirits. The estimation methodology is presented in section 3. In Section 4 we estimate the two specifications of the model by the (S)MM approach and give an economic interpretation of the empirical results. Finally, section 5 concludes. All relevant technical details are collected in the Appendix.

2 The Model: Rational Expectations versus Bounded Rationality

The baseline hybrid three-equations NKM reads as follows:

$$y_t = \frac{1}{1+\chi}\tilde{E}_t^j y_{t+1} + \frac{\chi}{1+\chi}y_{t-1} - \tau(\hat{r}_t - \tilde{E}_t^j \hat{\pi}_{t+1}) + \varepsilon_{y,t} \quad (1)$$

$$\hat{\pi}_t = \frac{\nu}{1+\alpha\nu}\tilde{E}_t^j \hat{\pi}_{t+1} + \frac{\alpha}{1+\alpha\nu}\hat{\pi}_{t-1} + \kappa y_t + \varepsilon_{\hat{\pi},t} \quad (2)$$

$$\hat{r}_t = \phi_{\hat{r}}\hat{r}_{t-1} + (1-\phi_{\hat{r}})(\phi_{\hat{\pi}}\hat{\pi}_t + \phi_y y_t) + \varepsilon_{\hat{r},t} \quad (3)$$

where the superscript $j = \{\text{RE, BR}\}$ refers to the rational expectations (RE) and the bounded rationality (BR) model, respectively. The corresponding expectations operator is $\tilde{E}_t^j$, which has to be specified for both models. It goes without saying that all variables are given in quarterly magnitudes.

In equation (1), the hybrid dynamic IS curve results from inter-temporal optimization of consumption and saving, which leads to consumption smoothing. The parameter $\tau \geq 0$ denotes the inverse inter-temporal elasticity of substitution in consumption behavior. Equation (2) represents the hybrid NKPC where the output gap (y_t) acts as the driving force behind inflation dynamics from monopolistic competition and Calvo-type sticky prices. The slope of the Phillips Curve is given by the parameter $\kappa \geq 0$. ν measures the discount factor ($0 < \nu < 1$). According to the Taylor rule with interest rate smoothing (equation (3)), the nominal interest gap is a predetermined variable, while the monetary authority reacts directly to contemporaneous movements in the output ($\phi_y \geq 0$) and inflation ($\phi_{\hat{\pi}} \geq 0$) gap. We account for intrinsic persistence in the stylized version of the well-known Smets and Wouters (2003, 2005 and 2007) model where backward-looking behavior is indicated by the parameters for habit formation χ, price indexation α and interest rate smoothing $\phi_{\hat{r}}$, respectively ($0 \leq \chi \leq 1$, $0 \leq \alpha \leq 1$, $0 \leq \phi_{\hat{r}} \leq 1$). We assume that the exogenous driving forces in the model variables follow idiosyncratic shocks $\varepsilon_{z,t}$, which are independent and identically distributed around mean zero and variance σ_z^2 with variables $z = \{y, \hat{\pi}, \hat{r}\}$.

Note here that we consider the gaps instead of the levels and therefore account explicitly for a time-varying trend in the inflation rate and the natural rate of interest. The corresponding gaps are simply given by taking the difference of the actual value for the output, the inflation rate and the nominal interest rate from their trends (i.e. time-varying steady state values) respectively, where the latter are computed by applying the Hodrick-Prescott filter with a standard value of the corresponding smoothing parameter of $\lambda = 1600$. Accordingly, the set of equations is used to describe the dynamics in the output gap y_t, the inflation gap $\hat{\pi}_t$ and the nominal interest rate gap $\hat{r}_t$, where $\hat{x}_t$ with $x = \{\pi, r\}$ denotes the deviations in both variables from the time-varying trend explicitly.[3]

[3]The results of previous studies show that the model may produce misleading results if a constant trend (e.g. a zero-inflation steady state) is assumed. For example, Ascari and Ropele (2009) observe that the stability of the economic system can depend on the variation

To make the description of the expectation formation processes more explicit, we examine two polar cases in the theoretical model framework of the baseline NKM. First, under rational expectations, the forward-looking terms are described by the expectations of the output gap and inflation gap at time $t+1$ in the equations (1) and (2):

$$\tilde{E}_t^{RE} y_{t+1} = E_t y_{t+1} \tag{4}$$
$$\tilde{E}_t^{RE} \hat{\pi}_{t+1} = E_t \hat{\pi}_{t+1}, \tag{5}$$

where E_t denotes the expectation operator conditional on information at time t. Second, as regards the other specification, we depart from rational expectations by considering the behavioral model of De Grauwe (2011). It is generally assumed that agents may adopt either an optimistic or pessimistic attitude towards movements in the future output gap (in the following indicated by the superscripts O and P, respectively):

$$E_t^O y_{t+1} = d_t \tag{6}$$
$$E_t^P y_{t+1} = -d_t, \tag{7}$$

where

$$d_t = \frac{1}{2} \cdot [\beta + \delta \lambda_{y,t}]. \tag{8}$$

Note here that the term d_t can be interpreted as "the divergence in beliefs among agents about the output gap" (De Grauwe (2011, p. 427)). The main difference between the RE and BR model is that the bounded rational agents are uncertain about the future dynamics of the output gap and therefore predict a fixed value of y_{t+1} measured by $\beta \geq 0$. We can interpret the latter as the *predicted subjective mean value* of y_t. However, this kind of subjective forecast is generally biased and therefore depends on the volatility in the output gap, i.e. given by the unconditional standard deviation $\lambda_{y,t} \geq 0$. In this respect, the parameter $\delta \geq 0$ measures the *degree of divergence* in the movement of economic activity. Note that due to the symmetry in the divergence in beliefs, optimists expect that the output gap will differ positively from the steady state value (which for consistency is set to zero), while pessimists will expect a negative deviation by the same amount. The value of δ remains the same with different types of agents.

in trend inflation. Cogley and Sbordonne (2008) also provide evidence for the explanation of inflation persistence by considering a time-varying trend in inflation. In the same vein, we regard the assumption of a constant natural rate of interest empirically unrealistic. In this paper, we follow the empirical approaches proposed by Cogley et al. (2010), Castelnuovo (2010) among others, who consider gap specifications for the inflation (and the nominal interest) rate. Furthermore, inflation and money growth are likely to be non-stationary in the Euro Area data. Given the presence of nonstationarity, we suggest that the estimation methodology such as the (S)MM approach presented here (or GMM in general) will lead to biased estimates of the model parameters. See also Russel and Banerjee (2008) as well as Assenmacher-Wesche and Gerlach (2008) among others for methodological issues related to non-stationary inflation in the US and Euro Area. Hence, in this study, we consider the gaps rather than the levels in order to ensure the behavior of the stationary times series.

The expression for the *market forecast* regarding the output gap in the bounded rationality model is given by

$$\tilde{E}_t^{BR} y_{t+1} = \alpha_{y,t}^O \cdot E_t^O y_{t+1} + \alpha_{y,t}^P \cdot E_t^P y_{t+1} = (\alpha_{y,t}^O - \alpha_{y,t}^P) \cdot d_t, \tag{9}$$

where $\alpha_y^O + \alpha_y^P = 1$ holds. The probabilities α_y^O and α_y^P indicate a stochastic behavior of the agents when choosing a particular forecasting rule (i.e. equation (6) or (7)). Note that α_y^O (or α_y^P) can be interpreted as the probability being an optimist (or pessimist). In the following, we give an explicit description of these probabilities. First, the selection of the forecasting rules (6) or (7) depends on the *forecast performances* of optimists and pessimists U_t^k (with $k = O,\ P$) given by the mean squared forecasting error. The utility for the forecast performances can be simply updated in every period as (cf. Brock and Hommes (1997)):

$$U_t^k = \rho U_{t-1}^k - (1-\rho)(E_{t-1}^k y_t - y_t)^2, \tag{10}$$

where the parameter ρ is used to measure the memory of agents $(0 \leq \rho \leq 1)$. Here $\rho = 0$ suggests that agents have no memory of past observations, while $\rho = 1$ means that they have infinite memory instead. Second, by applying the discrete choice theory under consideration of the forecast performances, agents can revise their expectations in which different performance measures will be utilized for $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$:[4]

$$\alpha_{y,t}^O = \frac{\exp(\gamma U_t^O)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} \tag{11}$$

$$\alpha_{y,t}^P = \frac{\exp(\gamma U_t^P)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} = 1 - \alpha_{y,t}^O, \tag{12}$$

where the parameter $\gamma \geq 0$ denotes the intensity of choice: if $\gamma = 0$, the self-selecting mechanism is purely stochastic ($\alpha_{y,t}^O = \alpha_{y,t}^P = 1/2$), whereas if $\gamma = \infty$, it is fully deterministic ($\alpha_{y,t}^O = 0,\ \alpha_{y,t}^P = 1$ or vice versa (see De Grauwe (2011), p. 429)). In other words, in the polar case where $\gamma = 0$ holds, agents are indifferent in being optimist or pessimist. Their expectation formation processes are independent of their emotional state when $\gamma = \infty$ holds, i.e. as they react quite sensitively to infinitesimal changes in their forecast performances.

We explain this revision process as follows. Given the past value of the forecast performance (U_{t-1}^k), we see that the lower the difference between the expected value of the output gap (taken from the previous period, i.e. $E_{t-1}^k y_t = |d_{t-1}|$) and its realization in period t, the higher the corresponding forecast performance U_t^k will be. More precisely, if e.g. the forecast made by the optimists is more accurate than the one made by the pessimists, this will lead to a higher level of utility for the optimistic agents, i.e. $U_t^O > U_t^P$ holds. Hence, the pessimists have the incentive to adopt the forecasting rule used by the optimists, which we can express as $E_t^O y_{t+1} = d_t$. Finally, this forecasting rule prevails and the share of pessimists in the market decreases. According to the equations (10) to (12), we can rationalize equation (9) by using simple substitution.

[4] See also Westerhoff (2008, p. 199) and Lengnick and Wohltmann (2013) among others for an application of discrete choice theory to models in finance and macroeconomics.

This results in a higher degree of volatility in the expectation formation process regarding the output gap relative to the outcome in the RE model – hereby we refer to Figure 1 given in section 4.3 for a clarification.

The same logic can be applied for the inflation gap expectations. Following De Grauwe (2011, pp. 436), we assume that agents will be either so called *inflation (gap) targeters* (tar) or *extrapolators* (ext).[5] In the former case, the central bank anchors expectations by announcing a target for the inflation gap $\bar{\hat{\pi}}$. From the view point of the inflation targeters, we consider this pre-commitment strategy to be fully credible. Hence the corresponding forecasting rule becomes

$$E_t^{tar}\hat{\pi}_{t+1} = \bar{\hat{\pi}}, \tag{13}$$

where we assume $\bar{\hat{\pi}} = 0$.[6] The extrapolators instead will expect that the future value of the inflation gap is given by its past value:

$$E_t^{ext}\hat{\pi}_{t+1} = \hat{\pi}_{t-1}. \tag{14}$$

Note that the market forecast for the inflation gap is similar to the forecast in equation (9):

$$\tilde{E}_t^{BR}\hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar}E_t^{tar}\hat{\pi}_{t+1} + \alpha_{\hat{\pi},t}^{ext}E_t^{ext}\hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar}\bar{\hat{\pi}} + \alpha_{\hat{\pi},t}^{ext}\hat{\pi}_{t-1}. \tag{15}$$

The forecast performances of inflation targeters and extrapolators are given by the mean squared forecasting error written as

$$U_t^s = \rho U_{t-1}^s \; - \; (1-\rho)(E_{t-1}^s\hat{\pi}_t - \hat{\pi}_t)^2, \tag{16}$$

where $s = (tar, ext)$ holds. Finally, we can write:

$$\alpha_{\hat{\pi},t}^{tar} \;=\; \frac{\exp(\gamma U_t^{tar})}{\exp(\gamma U_t^{tar}) + \exp(\gamma U_t^{ext})} \tag{17}$$

$$\alpha_{\hat{\pi},t}^{ext} \;=\; \frac{\exp(\gamma U_t^{ext})}{\exp(\gamma U_t^{tar}) + \exp(\gamma U_t^{ext})} = 1 - \alpha_{\hat{\pi},t}^{tar}, \tag{18}$$

where $\alpha_{\hat{\pi},t}^{tar}$ denotes the probability to be an inflation targeter. Economic agents will adopt a target behavior if the forecast performance using the announced inflation gap target is superior to the extrapolation of the inflation gap expectations, and vice versa. Note here that the memory (ρ), as well as the intensive of choice (γ), do not differ across the expectation formation processes in terms of the output and inflation gap. In the end, the BR model exhibits purely backward-looking behavior (cf. the equations (10) and (16)), while both forward- and backward-looking elements are contained in the RE model.[7] The

[5] This concept of behavioral heterogeneity has been widely used in financial market models, see e.g. Chiarella and He (2002) as well as Hommes (2006) among others.

[6] In this respect (based on an optimal monetary policy strategy), an inflation *gap* target of zero percent implies that the European central bank seeks to minimize the deviation of its (realized) target *rate* of inflation from the corresponding time-varying steady state value. Thus the deviation should be zero in the optimum.

[7] Note that the two models are overlapping, i.e. the forward-looking behavior in the RE model cannot be nested in the group behavior as being specified in a bounded rationality model. In particular, the expectation formation processes in both models do not share common structural features due to discrete choice theory.

solution to both systems can be computed numerically by backward-induction as well as the method of undetermined coefficients together with the brute force iteration procedure (Binder and Pesaran (1995)). However, the BR model does not have a simple closed-form solution. We refer to the Appendix A for details.

Finally, it can be argued that the BR model presented is not suitable for e.g. policy analysis since it is not based completely on microfoundations. In other words, the expectation mechanisms are imposed ex post on a system of structural equations which themselves have been derived from maximizing behavior under the assumption of rational expectations. However, the assumption for the divergence in beliefs (which reflects guessing) and the existence of the extrapolators can be guided by research evidence in experimental economics. In this respect it might be interpreted as pattern-based time-series forecasting rule done by De Grauwe (2011) and adopted in our study.[8]

From a theoretical point of view, Branch and McGough (2009) investigate the effects of heterogeneous expectations on a New-Keynesian framework where the forward-looking expressions in the dynamic IS curve and the NKPC are convex combinations of backward- and forward-looking behavior. The authors show that a micro-founded NKM under bounded rationality can be theoretically consistent if specific axioms are considered within the optimizing behavior of households and firms. These axioms lay the ground work for the ability of agents to forecast future realizations of the output gap and inflation (gap) at the micro level as well as aggregated behavior at the macro level. In comparison, De Grauwe (2011) allows for a switching mechanism based on discrete choice theory. It is an open question whether the latter based on the axioms by Branch and McGough (2009) may mitigate the (neglected) problems of misspecification. To sum up, there is no doubt that an extensive elaboration on the microfoundations of expectation formation is needed. However, cognitive information processing in the human brain remains ambiguous (De Grauwe (2011, p. 428, fn. 4)).

3 The Simulated Method of Moments Approach

In this paper we seek to match the model-generated autocovariances of the output gap, the inflation gap and the nominal interest rate gap with their empirical counterparts. Statistical inference on the market behavior is based on those model parameter values. The parameter estimates are considered as the result of the minimization of the distance between the model-generated and empirical second moments when applying the (S)MM approach. As mentioned in the Introduction, we focus on specific aspects of the data generating process when taking the models empirically to the real world. In other words, our study aims to show that a set of auto- and cross-covariances can be used to describe

[8]Roos and Schmidt (2012) find evidence for a backward-looking behavior in forming expectations by non-professionals in economic theory and policy. In their experimental study, they show that the projections of the future realizations in the output gap and the inflation rate are based either on historical patterns of the time series or - in the case of no information available - on simple guessing.

the regular properties of the business cycle.[9]

To be more specific, we suggest that the moment conditions play an important role in accounting for distributional properties of empirical data X_t with $t = 1, \cdots, T$, where T denotes the sample size. The sample covariance matrix at lag k is defined by:

$$m_t(k) = \frac{1}{T} \sum_{t=1}^{T-k} (X_t - \bar{X})(X_{t+k} - \bar{X})', \tag{19}$$

where $\bar{X} = (1/T) \sum_{t=1}^{T} X_t$ is the vector of the sample mean. The sample average of discrepancy between the model-generated and empirical moments is denoted as

$$g(\theta; X_t) \equiv \frac{1}{T} \sum_{t=1}^{T} (m_t^* - m_t), \tag{20}$$

where θ denotes a $l \times 1$ vector of unknown structural parameters. m_t^* and m_t are the *empirical* and the *model-generated* moment function, respectively (cf. equation (19)). In the empirical analysis of the structural model, an explicit closed-form solution for the endogenous variables in the RE model is used to derive the moment conditions (see Appendix A and Franke et al. (2012) for the intermediate steps needed). In the BR case, due to its non-linear structure, the analytic solution for m_t cannot be easily obtained. Therefore, the SMM approach must be applied instead (see further below). However, before we turn to the description of SMM, we briefly explain (again) the MM approach with respect to the estimation of the linear RE model.

One of main goals in this study is to draw inference from the underlying model to the auto- and cross-covariances of the observations at a (fixed) lag k with $k = 0, \cdots, n$.[10] After selecting an appropriate number of j variables for the lag length, we compute the corresponding p-dimensional vector of (empirical and simulated) moment conditions by

$$p = p(k, j) = (j \cdot k - 1) \cdot j. \tag{21}$$

From this, we avoid the double counting at the zero lags in the cross relationships by subtracting the term $j \cdot k$ by one. To construct a confidence interval for the auto- and cross-covariance moments, we use the Delta method. Here

[9] Indeed, its approximation will be as efficient as the maximum likelihood approach when appropriate moments are selected (see also Canova (2007, his Chapter 5)). For example, the Gaussian distribution can be approximated by the first and second moments. In addition, if empirical observations do not follow a Gaussian process, information on higher moments will be effective in matching the data generating process. In our empirical application, however, the sample size of the macro data being considered is not sufficient enough to provide accurate estimates on the higher moments. Thus we decide to focus on the second moments for our current study only, while further statistical analysis on the, let's say, optimal selection of the moments being left for future research.

[10] The maximal number of lags (denoted by n) is chosen under consideration of the economic model and sample size. In the macroeconomic model being considered here, n captures the length of the business cycle.

we apply a linear approximation of the moment function around the point estimates under consideration of the corresponding gradient vector (see Appendix B for details).

Given that the duration of the business cycle lies between (roughly) one and eight years in the Euro Area (cf. Artis et al. (2003)), a reasonable compromise is a length of two years. According to equation (21), $p = 78$ moments are then considered as appropriate choice for our study, where we (based on the underlying model structure) take into account a lag length of $n = k^{max} = 8$ and $j = 3$ variables. As we choose a lag structure in the auto- and cross-covariances of $n = k^{max} = 8$, we take 1/4 of the total length of a business cycle into account, where the latter is defined by the change in the output gap from a trough to the next one. In the next section we will show that the output gap exhibits similar patterns in the upswing movement from a trough to a peak over the whole time series. Therefore we claim that the relevant observations rely on $p = 78$ moments only, since repeating patterns in the time series do not exhibit additional information.[11]

With a focus on these moment conditions, we can estimate the model parameters by minimizing the following quadratic objective function:

$$J(\theta) = \min_{\theta} \; g(\theta; X_t)' \, \widehat{W} \, g(\theta; X_t), \tag{22}$$

where more importance is attached to particular moment conditions according to the weighting matrix $\widehat{W}$ (see Andrews (1991)). The kernel estimator has the following general form with the covariance matrix of the appropriately standardized moment conditions given by

$$\widehat{\Gamma}_T(h) = \frac{1}{T} \sum_{t=h+1}^{T} (m_t - \bar{m})(m_t - \bar{m})', \tag{23}$$

where $\bar{m}$ once again denotes the sample mean. To find an appropriate lag length, we use a popular choice of $h \sim T^{1/3}$, that is, $h = 5$ for estimating the covariance matrix in the Euro Area (i.e. the Hansen-White covariance estimator):

$$\widehat{\Omega} = \widehat{\Gamma}_T(0) + \sum_{h=1}^{5} \Big(\widehat{\Gamma}_T(h) + \widehat{\Gamma}'_T(h) \Big). \tag{24}$$

The weighting matrix $\widehat{W}$ is computed from the inverse of the estimated covariance matrix $\widehat{\Omega}$. However, a high correlation between the moment conditions that we consider makes the estimated covariance matrix nearly singular. Singularity in the covariance matrix stands out, as the moment conditions and the elements of the weighting matrix are highly correlated when the small sample size is used (Altonji and Segal (1996)). To circumvent the econometric issues, we use the

[11] Broadly speaking, the business cycle can be seen approximately as a sinus function, where a fraction of 1/4 describes the upswing movement if a trough being the starting point. Hence, the transition from the peak to the second trough mimics the upswing with, of course, the opposite sign. Therefore, we judge our choice of $p = 78$ moments (or, equivalently, a fraction 1/4 of the length of the business cycle) as being valid.

diagonal matrix entries as the weighting scheme, while the off-diagonal components of the matrix $\widehat{W} = \widehat{\Omega}^{-1}$ are ignored. Although the weighting matrix is not optimal enough to provide unbiased or consistent parameter estimates, we claim that this procedure is based on an economic (rather than a strict econometric) rationale. The estimated confidence bands (intervals), then, become wider, as the sandwich elements in the covariance of parameter estimates cannot cancel each other out in the presence of this weighting scheme (see also Anatolyev and Gospodinov (2011)).[12]

Next, to make inferences about the model parameters, we study the properties of the sample distribution for the parameter estimation. In particular, under certain regularity conditions, we arrive at the following asymptotic distribution of the model parameters:

$$\sqrt{T}(\widehat{\theta}_T - \theta_0) \sim N(0, \Lambda), \tag{25}$$

where $\Lambda = [(DWD')^{-1}]D'W\Omega WD[(DWD')^{-1}]'$ holds. D is the gradient vector of moment functions evaluated around the point estimates. This can be written as:

$$\widehat{D} = \left.\frac{\partial m(\theta; X_T)}{\partial \theta}\right|_{\theta=\widehat{\theta}_T}. \tag{26}$$

Under RE, we can obtain the simple analytic moment conditions of the model as described above. However, for the *BR model*, the analytic expressions for the moment conditions are not available because of its non-linear structure and, in particular, discrete choice theory (see again Appendix A). To circumvent this problem, we use the simulated data to estimate the behavioral parameters in the BR model. In particular, SMM is suited to a situation where the model is easily simulated by replacing theoretical moments. Then the model-generated moments in equation (22) are replaced by their simulated counterparts:

$$m_t = \frac{1}{S}\sum_{s=1}^{S} \widetilde{m}_t. \tag{27}$$

In equation (27), we approximate the theoretical moments (m_t) based on the simulated data of $\widetilde{m}_t$. The simulation size is denoted by S. Under certain regularity conditions, the SMM estimator is known to be asymptotically normal (Duffie and Singleton (1993), Lee and Ingram (1991)):

$$\sqrt{T}(\widehat{\theta}_{SMM} - \theta_0) \;\sim\; N(0, \Lambda_{SMM}), \tag{28}$$

where $\Lambda_{SMM} = [(B'WB)^{-1}]B'W\ (1 + 1/S)\ \Omega\ WB[(B'WB)^{-1}]'$ holds, that is a covariance matrix of the SMM estimates. A gradient vector of the moment function is defined as $B \equiv E\left[\left.\frac{\partial m_t}{\partial \theta}\right|_{\theta=\widehat{\theta}}\right]$.

[12] The possibility of a non-optimal weighting matrix can be identified as a drawback of the (S)MM approach. However, we state that the SMM can be successfully used to estimate non-linear bounded rational NKM in terms of the features given by transparency and muted manageability. Alternative estimation approaches like e.g. simple GMM, Indirect Inference, Bayesian techniques or Non-Linear Least Squares exhibit problems with respect to non-linear data generating processes, the use of auxiliary models, the requirements for sufficient prior information and Gaussian shocks, respectively.

However, the model estimation is now subjected to simulation errors, as the estimated covariance matrix is less accurate than its analytic counterpart. In particular, a linear approximation to the non-linear dynamics of the BR model (i.e. the Delta method) will be inaccurate.[13] To find the parameter values with certainty, we compute the standard errors by using the following steps:

1. The BR model is estimated using a simulation size of $S = 10$.
2. The estimation is iterated over 100 times, while different random seeds are used to obtain point estimates of the model parameters for each iteration.
3. We take 100 different estimates to compute the mean and standard error of the parameter estimates.

Indeed, the above iterative method corresponds to the case where we estimate the model based on a simulation size of 1,000. The iterative approach to the model estimation can be used to avoid large simulation errors; the parameter estimates based on simulation are likely to be deviated from the true values of the model parameters because of the non-linear switching rule being assumed. We provide an alternative approximation to the parameter values over each iteration with a small simulation size.[14]

Accordingly, we can obtain the simulated intervals for the model parameters, especially for the behavioral parameters with certainty. Finally, we use the J test to evaluate compatibility of the moment conditions:

$$\bar{J} \equiv T \cdot J(\widehat{\theta}) \overset{\mathrm{d}}{\to} \chi^2_{p-l}, \tag{29}$$

where l denotes the number of parameters to be estimated. Note that the J-statistic is asymptotically χ^2 distributed with $(p - l)$ degrees of freedom. In this study, the lag length for the covariance is set to two years. Hence, the number of moment conditions exceeds the model parameters, and we consider this particular case as overidentification.[15] Note that the degrees of freedom is smaller in the hybrid RE than the hybrid BR model (66 versus 68) due to the additional behavioral parameters, β and δ, to be estimated.

[13] We can reduce the approximation error by increasing the simulation size in order to maintain $\frac{1}{S} \to 0$. For instance, the simulation error becomes 1% when the moments are simulated 100 times. However, the non-linearity often increases the magnitude of changes in these errors, i.e. $E(\eta(\hat{\theta})) \neq \eta(E(\hat{\theta}))$ holds, where η is a highly non-linear function. Because of this, we do not use the Delta method to measure the uncertainty of parameter estimates on the BR model.

[14] Alternatively, Jang (2013) discussed the parameter uncertainty of a stochastic agent-based model by investigating the simulated parameter space.

[15] However, if the off-diagonal components in the estimated covariance matrix $\widehat{\Omega}$ (given in equation (24)) are discarded, the distribution in the J-statistic is likely to have a larger dispersion than the χ^2-distribution with degrees of freedom of $(p - l)$. Indeed, when the weighting matrix is non-optimal or some moment conditions are not valid, the J-statistic is no longer χ^2 distributed. To examine the effectiveness of the J-test, in a working paper version of this paper, Jang and Sacht (2012) investigate the validity of the weighting matrix with our chosen moment conditions via an extensive Monte Carlo study.

4 Empirical Application to the Euro Area

4.1 Data

Euro Area data set is retrieved from the 10th update of the Area-Wide Model (see Fagan et al. (2001)) quarterly database. The data applied in this study cover the period from 1975:Q1 to 2009:Q4. The output gap and interest rate gap are computed from real GDP and nominal short-term interest rate, respectively. A standard smoothing parameter of $\lambda = 1600$ is used to estimate the trend of the observed data from the Hodrick-Prescott filter. The inflation measure is the quarterly log difference of the Harmonized Index of Consumer Prices (HICP) instead of the GDP deflator. The inflation gap is also computed using the Hodrick-Prescott filter.[16] The sample for this data set is available from 1970:Q1 onwards. Five years are considered in a rolling window analysis to estimate the perceived volatility of the output gap $\lambda_{y,t}$. The underlying MATLAB codes are available upon readers request.

4.2 Basic Results

Table 1 presents the parameter estimates when the intensity of choice γ is varied over an admissible range, that is, $\gamma \in \{0.1, 1, 10, 100\}$ – where the value of 100 serves as an approximation to infinity. Hence, we cover the cases where the self-selecting mechanism is close to being purely stochastic ($\gamma = 0.1$) up to being almost fully deterministic ($\gamma = 100$). In addition, this parameter is set to unity, which is in line with De Grauwe (2011, p. 439) and account for a moderate degree in the intensity of choice together with a value of 10. We choose this treatment in order to check how sensitive the estimation results are with respect to the switching process across different forecasting strategies. We have difficulties in identifying the parameter γ because of the non-linear structure based on the discrete choice mechanism (see Appendix A). Several authors address these problems in their studies, e.g. Gaunersdorfer and Hommes (2007) as well as Goldbaum and Mizrach (2008).[17] Furthermore, in our application, the structural representation of the behavioral parameters can cause multiple optima over a particular parameter space, i.e. the point estimates are located in economically uninteresting and/or implausible regions. By calibrating γ we focus on our objective of the empirical application, that is, the economic interpretation of the model parameters in the BR model. This holds especially for those parameters which account for the existence of animal spirits besides γ — namely the predicted subjected mean value and the degree of divergence

[16] The HICP and GDP deflation can be used to measure the consumer price index (CPI) and producer price index (PPI) inflation, respectively. For our empirical application, inflation is measured by the HICP instead of the implicit GDP-deflator, since the former has been widely used for micro-level analysis. For instance, Forsells and Kenny (2004) show that inflation expectations can be approximated by micro-level data like consumer surveys (i.e. in the European Commission survey indicators). Also see Ahrens and Sacht (2014) for a discussion on using the HICP instead of the GDP-deflator in macroeconomic studies.

[17] Gaunersdorfer et al. (2008) show that the intensity of choice is a crucial parameter with respect to (local) stability in this kind of bounded rationality model with regime-switching.

given by β and δ, respectively. Nevertheless, we emphasize that the estimation of γ will be a fruitful project to be undertaken in future research.

De Grauwe emphasizes in his original paper that the "heuristic model does not need lags in the transmission process to generate inertia" (2011, fn. 11, p. 443). Although it can be agreed on this from a theoretical point of view, we show that this statement is empirically incorrect. To see this, we estimate a purely forward-looking version of the BR model where price indexation and habit formation are constrained, i.e. $\chi = \alpha = 0$ holds. The value of J (see second last row of Table 1) suggest that the fit of the forward-looking specification does not outperform the hybrid specification of the model. Whether this observation holds by increasing the number of deep parameters to be estimated (i.e. including the lags) in the latter case is not of primary concern. Most importantly, the backward-looking elements in the dynamic IS equation and the NKPC play indeed a distinct role in matching the empirical moments. Therefore we focus on the comparison of the hybrid RE to the hybrid BR model only.

In the hybrid BR case, we will consider those parameter estimates for interpretation, which are connected to the lowest value of the loss function given the corresponding value of γ. However, the values of the criterion function for the cases $\gamma = 0.1$ and $\gamma = 1$ do not differ, i.e. $J = 51.51$ vs. $J = 53.72$ holds. Given the corresponding p-values (0.905 vs. 0.861), both models provide almost an equal fit to the data. Hence, we interpret those point estimates based on the empirical evaluation in the case $\gamma = 1$. The reason is that the confidence interval for parameter estimates is narrow in this case - especially for the parameters α and β. For the latter, the increase in uncertainty is not really surprising as γ approaches zero because the switching process becomes almost purely stochastic in that case. As the intensity of choice is set close to its lower bound, expectations do not influence the realizations of the current output gap and inflation gap. Therefore we can conclude that the point estimate for β is hard to pin down. Since there is less uncertainty connected to the estimation results of the parameters as $\gamma = 1$ holds, however, we claim that the results are more reasonable for economic interpretation – besides the point estimates do not differ (except for β) either. In addition, the cases of strong switching processes ($\gamma = 10$ and $\gamma = 100$) do not provide a good approximation to the data generating process. According to the model specification test, the corresponding p-value is very small and they are rejected as being a 'true' model with a high probability.

As it is common in a persuasive amount of empirical studies, the discount parameter ν is calibrated to 0.99. And the memory parameter ρ is set to zero, i.e. past errors are not taken into account (cf. the equations (10) and (16)), since this is based on the empirical result for all our estimations. From this, we see that a strict forgetfulness or cognitive limitation holds, which is a requirement for observing animal spirits (cf. De Grauwe (2011, p. 440)).[18] By fixing those parameters in the final estimation, we may partially alleviate the problems with

[18] For clarification, we apply the estimations for all calibrated values of γ and include ρ as a parameter to be estimated. Hence, the parameter ρ is equal to zero in all cases.

a high-dimensional parameter space, and focus on a particular subspace which will be more interesting for (empirical) economists. Given these assumptions, we can separately obtain the estimates for remaining parameters from the RE and BR model via (S)MM. Note here that the Taylor principle $\phi_{\hat{\pi}} > 1$, which is required to hold in order to ensure the stability of the system, is always fulfilled. For that purpose, we set the lower bound of this monetary policy parameter equal to one.

4.3 Economic Interpretation of the Results

Several observations are worth mentioning. The parameter estimate for the degree of price indexation α is higher in the BR (0.973) than the RE (0.765) model. It follows that the expressions, which are in front of the forward- and backward-looking terms in the Phillips Curve, indicate a (slightly) higher weight on future inflation $\tilde{E}_t^j \hat{\pi}_{t+1}$ (i.e. $\frac{\nu}{1+\alpha\nu} > \frac{\alpha}{1+\alpha\nu}$), where the result is more pronounced for the RE ($0.563 > 0.437$) than the BR ($0.504 > 0.496$) model. From this, we see that there is strong evidence for a hybrid structure of the NKPC in both models. As mentioned earlier, however, the forward-looking behavior in the BR model is controlled via the group dynamics based on the forecasting performance given in Equation (10). The empirical applications of the BR model show that the dynamics of the inflation gap are driven by the expectation formation process (i.e. the evaluation of the forecast performance) in the inflation gap, i.e. agents' cognitive limitation. In other words, we find evidence for a backward-looking expectation formation process, since the estimated value for α is quite high: one group believes in a central bank inflation target of zero percent (equation (13)) while the other group of agents form their expectations in a purely *static* way (equation (14)), i.e. under consideration of $\hat{\pi}_{t-1}$. More precisely, the group dynamics with respect to the price setting scheme are not sufficient enough to account for the persistence in the inflation gap alone.

Furthermore, the results on the estimate of price indexation have important implications for conducting optimal monetary policy. According to Leitemo (2008), the optimal targeting rule in the (pre-)commitment case has to be much more forward-looking the higher the degree of backward-looking behavior in the NKPC will be. Our results indicate that this most likely holds for the RE model via MM. But the result is quite different when the Bayesian technique is applied; i.e., the parameter of price indexation is estimated to be zero (cf. Smets and Wouters (2003, 2007), Benati and Surico (2009), Cogley et al. (2010) among others). In particular, the estimated high degree of price indexation will have a similar impact on the results from optimal monetary policy analysis (as reported in Leitemo (2008)) based on the BR model, although such kind of investigation has not to be undertaken (to the best of our knowledge) in the literature so far.

Regarding the dynamic IS equation in the RE model, the output gap is influenced by the same proportion of forward- and backward-looking behavior, since the empirical result shows that $\chi = 0.999$ holds. Any specific statement cannot be made with respect to the impact of habit formation on the dynamics of the output gap in the BR model, since χ is estimated to be insignificant

Label	hybrid RE	$\gamma = 0.1$		$\gamma = 1$		$\gamma = 10$		γ=100	
		pure BR	hybrid BR	pure BR	hybrid BR	pure BR	hybrid BR	pure BR	hybrid BR
α	0.765	**0**	0.931	**0**	0.973	**0**	0.856	**0**	0.432
	0.481 - 1.000		0.657 - 1.000		0.831 - 1.000		0.440 - 1.000		0.000 - 0.947
κ	0.035	0.381	0.191	0.312	0.175	0.355	0.178	0.268	0.269
	0.011 - 0.058	0.294 - 0.468	0.140 - 0.242	0.219 - 0.404	0.139 - 0.211	0.106 - 0.603	0.071 - 0.284	0.0 - 0.575	0.117 - 0.421
$\sigma_{\hat{\pi}}$	0.275	0.446	0.495	0.297	0.498	0.192	0.432	0.164	0.277
	0.097 - 0.453	0.105 - 0.788	0.269 - 0.721	0.000 - 0.643	0.326 - 0.670	0.000 - 0.408	0.084 - 0.780	0.000 - 0.370	0.000 - 0.577
χ	0.999	**0**	0.547	**0**	0.451	**0**	0.542	**0**	0.351
	0.349 - 1.000		0.000 - 1.000		0.000 - 1.000		0.000 - 1.000		0.000 - 0.842
τ	0.079	0.737	0.329	0.371	0.322	0.179	0.259	0.164	0.242
	0.000 - 0.222	0.298 - 1.177	0.106 - 0.551	0.172 - 0.570	0.158 - 0.486	0.000 - 0.663	0.026 - 0.492	0.000 - 0.442	0.000 - 0.559
σ_y	0.561	0.423	0.669	0.808	0.793	0.760	0.835	0.635	0.756
	0.354 - 0.768	0.018 - 0.827	0.364 - 0.974	0.654 - 0.962	0.630 - 0.957	0418 - 1.102	0.623 - 1.048	0.204 - 1.065	0.490 - 1.021
$\phi_{\hat{\pi}}$	1.289	1.527	1.128	1.816	1.145	1.316	1.250	1.322	1.284
	1.000 - 1.944	1.000 - 2.743	1.000 - 1.534	1.000 - 3.313	1.000 - 1.485	1.000 - 2.059	1.000 - 1.973	1.000 - 2.031	1.000 - 1.826
ϕ_y	0.497	0.621	0.681	0.713	0.674	0.580	0.608	0.755	0.589
	0.058 - 0.936	0.257 - 0.985	0.438 - 0.924	0.252 - 1.175	0.443 - 0.904	0.066 - 1.094	0.239 - 0.978	0.000 - 1.911	0.181 - 0.997
$\phi_{\hat{r}}$	0.604	0.685	0.675	0.739	0.677	0.602	0.660	0.488	0.578
	0.411 - 0.797	0.614 - 0.755	0.613 - 0.736	0.671 - 0.808	0.608 - 0.746	0.305 - 0.900	0.534 - 0.786	0.075 - 0.901	0.292 - 0.863
$\sigma_{\hat{r}}$	0.421	0.109	0.216	0.269	0.256	0.439	0.304	0.435	0.361
	0.140 - 0.701	0.000 - 0.409	0.000 - 0.563	0.000 - 0.601	0.000 - 0.565	0.000 - 0.965	0.000 - 0.728	0.000 - 1.020	0.000 - 0.840
β	-	5.563	4.584	2.379	2.269	3.101	2.274	1.645	1.927
		3.899 - 7.227	1.918 - 7.249	1.417 - 3.341	1.082 - 3.456	0.0 - 7.767	0.000 - 6.331	0.387 - 2.902	0.565 - 3.289
δ	-	0.709	0.688	0.379	0.488	1.291	0.465	0.658	0.386
		0.000 - 2.123	0.000 - 2.789	0.000 - 1.278	0.000 - 1.653	0.000 - 4.455	0.000 - 1.736	0.000 - 2.655	0.000 - 1.403
J	56.31	80.04	51.51	100.88	53.72	100.52	71.67	241.61	122.93
p	0.843	0.151	0.905	0.006	0.861	0.006	0.295	0.000	0.000

Table 1: Estimation results (RE versus BR model).

Note: We consider $p = 78$ moments (two years), based on the SMM approach. The 95% confidence interval is given in brackets. The degrees of freedom for the χ^2 distribution amount to 68 (hybrid RE, pure[ly forward-looking] BR) and 66 (hybrid BR). The 5% critical values are 88.25 and 85.96, respectively. No memory is assumed in the BR models ($\rho = 0$) due to pre-estimations. The discount factor ν is calibrated to 0.99. The p-value is denoted by p.

in this case. This result indicates that there is no possible interpretation for the role of the habit formation process in explaining the persistence in the output gap. In other words, inertia seems to be fully captured by the switching process with respect to the future output gap realizations. Furthermore, the result on the interest rate smoothing parameter supports a moderate degree of persistence ($\phi_{\hat{r}} = 0.677$) in the nominal interest rate gap in the BR model. This observation is consistent with the results obtained from Bayesian estimations taken from the literature. In our study, however, the results obtained by MM show that the point estimate for $\phi_{\hat{r}}$ is less pronounced.

In addition, the empirical estimates for κ and τ in the RE model provide an evidence of a small degree of inherited persistence, where the latter measures the cross-relationships between the output and inflation gap. These results suggest that both economic indicators respond less to the changes in the associated driving forces, i.e. the real interest rate gap and the output gap, respectively. However, this does not hold for the BR model. In particular, the changes in the output gap have a strong impact on the movements in the inflation gap (indicated by $\kappa = 0.175$) compared to the RE case ($\kappa = 0.035$). For the output gap in the BR model, inherited persistence plays a fundamental role in shaping the dynamics of this variable, which can be seen through the high values of inverse inter-temporal elasticity of substitution ($\tau = 0.322$). For the RE model, this parameter is estimated to be insignificant. This implies that bounded rational agents are likely to have a higher degree of risk aversion than the representative agents. To sum up, our results show that in the BR model, the cross-movements in the output and inflation gap account for the persistence in both variables (under consideration of perfect habit formation $\chi = 0.999$ in the RE case) rather than price indexation alone. This can be seen through the high values for κ and τ together with α in which the model allows for a limited cognitive ability of agents.

The variance in the output and inflation gap shocks are estimated to be larger for the BR ($\sigma_y = 0.793$ and $\sigma_{\hat{\pi}} = 0.498$) than those of the RE ($\sigma_y = 0.561$ and $\sigma_{\hat{\pi}} = 0.275$) model, respectively. The results reveal that the volatility of the output and inflation gap are closely related to the switching rules with respect to the consumption and price-setting behavior. For instance, the waves of optimism and pessimism act much like a persistent force in the output gap fluctuations going from peaks to troughs. Figure 1 illustrates that the peak of the fluctuation in the simulated output gap for the BR case (middle-left panel) corresponds to the market optimism (lower-left panel) and vice versa. The qualitative interpretation remains almost the same for the inflation gap in the BR case (middle- and lower-right panels, respectively) – but the dynamics of extrapolators are highly volatile reflecting the large variations in the second moments of the empirical inflation gap (upper-right panel). Furthermore, whether $\gamma = 1$ is a high, moderate, or low intensity of choice depends also on the variance of the forecast performance given by U^i with $i = \{k, s\}$. This can be seen from both bottom panels of Figure 1. In the case of the output gap, the switching between regimes is clearly more 'aggressive' than for the inflation gap, despite the intensity of choice being fixed at $\gamma = 1$ for both variables.

The fit of the models should not be directly compared by illustrating the simulated time series (middle-panels) as the simulated ones display a single realization of the stochastic processes from the model. However, we see that the series qualitatively resembles their empirical counterparts (upper-panels). Finally, the nominal interest rate shocks $\sigma_{\hat{r}}$ in the RE model are estimated to be 0.421, while in the BR case no interpretation can be made since the point estimate is not significant.

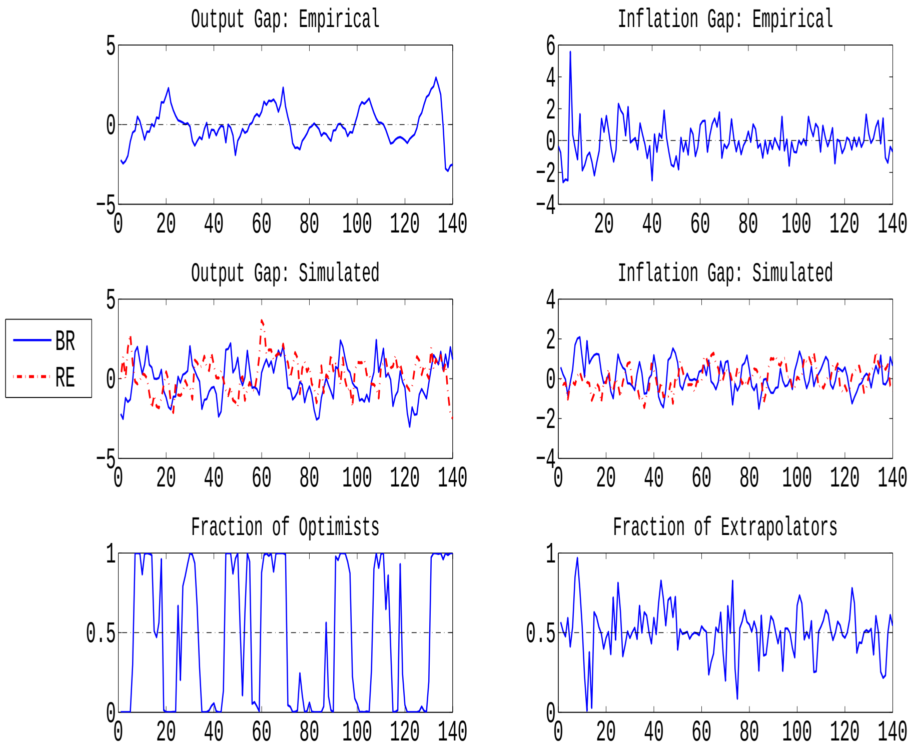

Figure 1: Dynamics in the output and inflation gap (RE versus BR model).

Note: The upper and middle panels display the empirical and simulated values for the output gap (left) and the inflation gap (right), while the lower panels display the corresponding fractions of market optimists (left) and extrapolators (right). The simulated time series are computed using the point estimates for the parameters in the RE and BR model in Table 1. The time in quarters is displayed on the horizontal axis.

The remaining parameter estimates confirm the known results from the literature where the monetary authority may react slowly to changes in the output gap, while the opposite holds for the coefficient on the inflation gap (RE: $\phi_y = 0.497$ and $\phi_{\hat{\pi}} = 1.289$ vs. BR: $\phi_y = 0.674$ and $\phi_{\hat{\pi}} = 1.145$). The results for $\phi_{\hat{\pi}}$ suggest that the Taylor principle clearly holds over the whole sample period when $\phi_{\hat{\pi}}$ is significantly higher than its lower bound of one. This is true in both cases. Nevertheless, the results for the BR model indicate a stronger concern in the output gap movements relative to the RE model. It is worth mentioning that the estimation results indicate that the monetary policy coefficient on the output gap ϕ_y is equal to 0.673, which is consistent with the observations by De Grauwe (2011, pp. 443-445). His simulations show that flexible inflation targeting can reduce both output gap and inflation (gap) variability at its minimum level when ϕ_y lies in the range between 0.6 and 0.8.

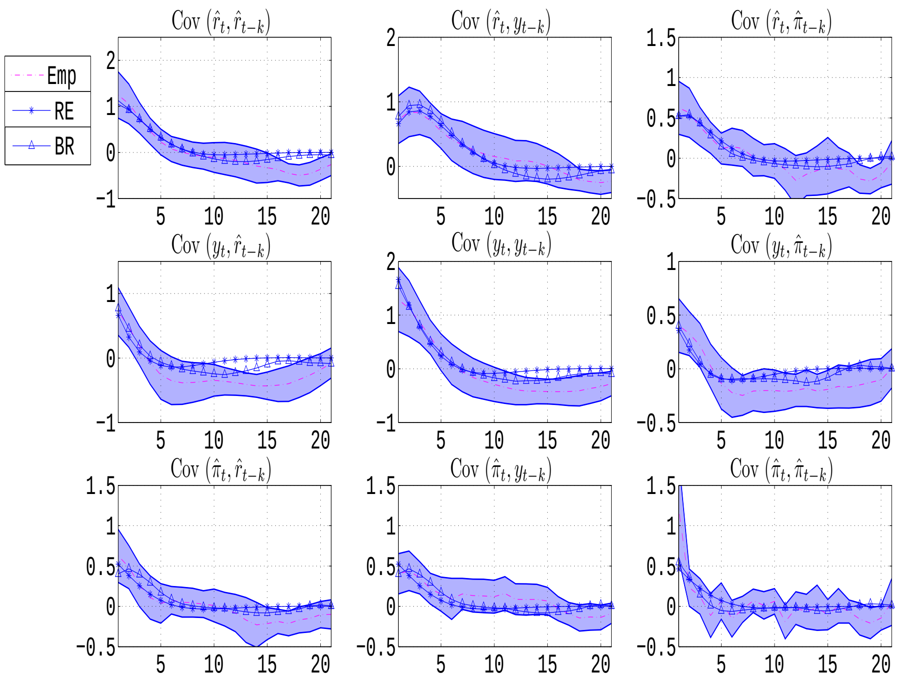

Figure 2: Model covariance (Cov) profiles in the Euro Area.

Note: The dashed line results from the empirical covariance estimates. The 95% confidence intervals for the empirical moments are represented by the shaded area. The triangle (BR) and star (RE) lines indicate the model generated ones. The confidence bands are computed via the Delta method (see Appendix B). The lag length is displayed on the horizontal axis.

The interpretation of this observation is manifold. First, we can consider the case of strict inflation targeting, where the central bank does not account for the volatility in the output gap.[19] As a result, the targeting rule does not change the forecast performance of the optimists and pessimists, as the real interest rate gap in the dynamic IS curve does not response directly to monetary policy. However, there is still an indirect effect (even highly volatile movements in y_t

[19]Note here, that strict and flexible inflation targeting are measured by low and high values of ϕ_y in the *Taylor rule*, respectively. This definition of inflation targeting is rather uncommon in the literature, while it is more connected to the corresponding weights in the *loss function* of the central bank in terms of optimal monetary policy. However, we follow the interpretation by De Grauwe (2011, pp. 443-445) to build a bridge from our empirical results to his theoretical analysis.

are not dampened by the policy makers) indicated by κ in the NKPC. Hence, due to the high degree of inherited persistence the strict inflation targeting cannot control fluctuations in the output and inflation gap. Second, in the case of strong output gap stabilization (relative to the inflation gap) the central bank dampens its pre-commitment to an inflation target. The amplification effects of this kind of policy on the forecast performances of the inflation extrapolators will then result in a higher volatility of the inflation gap. In this respect, it can be concluded that our empirical findings account for neither the first nor the second extreme case, but for a moderate degree of flexible inflation targeting in the Euro Area over the observed time interval instead.[20]

As mentioned earlier, a major emphasis in our study is to find the values of the bounded rationality parameters. First, over the whole sample period, the optimistic agents have expected a fixed divergence of belief indicated by $\beta = 2.269$. Roughly speaking, the optimists have been really optimistic that the future output gap will differ *positively* by slightly above one percent on average from its steady state value.[21] Due to the symmetric structure of the divergence in beliefs, pessimistic agents were moderately pessimistic over the same sample period. From their point of view the future output gap was expected to be around one percent on average *below* its steady state value. The point estimate for the bounded rationality parameter δ, which measures the divergence in beliefs, is *unfortunately* insignificant. Hence, one can not make any statements about how both types of agents felt confident about their expectations. In other words, there is no clear evidence for either a low or high degree of uncertainty connected to the expected future value of y_t given by β.

In addition, the value for $\rho = 0$ indicates endogenous and inherited persistence (α, χ, κ and τ). The highly subjective expected mean value of the output gap β - in conjunction with the dynamics induced by the self-selecting mechanisms (see the corresponding fractions in the lower-panels in Figure 1) - explains the (high) volatility of the output gap. According to the discrete choice theory, we see that this strengthens the optimistic agents' belief about the future output gap to diverge in the data, since they can over(or under)react to the underlying shocks that occur within the Euro Area. The same observation holds for the inflation gap dynamics. The proportion of the extrapolators in the economy corresponds to the empirical inflation gap movements (cf. lower right vs. upper-right panels in Figure 1): the higher the fraction of extrapolators is, the more volatile the inflation gap dynamics due to backward-looking expectations, i.e. under the past realizations $\hat{\pi}_{t-1}$ will be.

The visual inspection in Figure 2 shows a *fairly remarkable* fit of the models to the data. The match of both models looks clearly good over the first few lags and still fairly good over the higher lags until a lag of two years. Ex-

[20] Indeed, there are a plethora of studies on the estimation of (small, medium or large) *linear* NKMs with *rational expectations* using Euro Area data, e.g. Smets and Wouters (2003) and Moons et al. (2007) among others. To the best of our knowledge, however, some of these studies provide some implications which differ from our contribution with respect of the use of the GMM and the Bayesian approach and non de-trended specifications of $\hat{\pi}_t$ and $\hat{r}_t$.

[21] Note that the expected future value of the output gap is given by $E_t^i y_{t+1} = |d_t| = \frac{1}{2}\beta$ *on average* with $i = \{O, P\}$.

ceptions can be found in the $(y_t, \hat{r}_{t-k})$, $(y_t, \hat{\pi}_{t-k})$ and $(\hat{\pi}_t, y_{t-k})$ nexus, where the simulated covariance profiles generated by both models diverge from their empirical counterparts beginning at around a lag of one year. More precisely, the simulated covariance profiles of the RE model approximate the empirical ones for the cross-covariance $(\hat{\pi}_t, \hat{r}_{t-k})$ quite well, while the same holds for the BR model in case of the cross-/auto-covariances $(\hat{\pi}_t, y_{t-k})$ and $(\hat{\pi}_t, \hat{\pi}_{t-k})$. In the latter, although the sharp decline in the autocovariance of the inflation gap over the first two lags is not covered by both models, the BR specification matches the empirical profiles *slightly better* than the RE one. In any case, all of the moments are *inside* the confidence intervals of the empirical moments up to lag 8. The graphical results are also confirmed by the values of the objective function J for the RE (56.31) and BR (53.72) model in the second to last row of Table 1. The asymptotic χ^2 distributions for the J-test have the degrees of freedom of 68 and 66 for the RE and BR model, respectively. Since the critical values at 5% level are 88.25 and 85.96, that is the estimated loss function values are smaller than these criteria, we do *not* reject the null hypothesis that these models can approximate the data generating process well. Moreover, the simulated trajectory shows a *remarkable fit of the BR model*, which leads to some confidence in the estimation procedure. It can be concluded that the bounded rationality model presented here can provide a reasonable fit to the empirical auto- and cross-covariances.

Note that the significant differences between the two models could be tested using a formal model comparison method, since the models do not have any difficulties to fit the empirical moments at the 5% significant interval (see also Jang (2012) among others). In other words, the J-test only evaluates the validity of the model along the lines of the chosen moment conditions. Therefore we cannot provide a direct comparison between the fits of the two models. More rigorous tests will be a priority in future research. Nevertheless, our empirical results indicate that the empirical test on bounded rationality (i.e. the assumption of the divergence in beliefs) has to be treated carefully, because some parameters (including the behavioral parameter δ) within the non-linear modeling approach are *insignificant.*

While the current study focused on Euro Area data in which we could use the large sample size for parameter estimation, an estimation for e.g. the US economy would be a promising exercise to be undertaken. However, note that in this case the sample size must be split into two sub-periods namely the Great Inflation and Moderation period, respectively. This is necessary due to the existing structural breaks in the US time series based on a change in inflation's volatility from high to low at beginning of the 1980s. We apply a *robustness check*, where we re-estimate both models based on *US data* provided by the St. Louis FED (in gap specification) for the sub-periods 1960:1 – 1979:2 (Great Inflation period) and 1982:4 – 2007:2 (Great Moderation period) as well as for the whole sample 1960:1 – 2007:2. In each of these cases we found that the RE model outperforms the BR one given the associated p-value. In particular, the model has difficulties to explain the high volatility in interest rate and inflation gap over the whole sample period, while we state that because of the small sample size in both sub-periods for the US the BR model is hard to evaluate.

5 Conclusion

In this paper, we provide empirical evidence for a non-linear expectation formation process as well as the relevance of bounded rational acting agents via SMM. In particular, the validity of the model assumptions on the cognitive limitation (e.g. because of different individual emotional states) is empirically tested using historical Euro Area data. To show this, we focus on the estimation of the behavioral parameters in the model, which can account for the amplification channel from animal spirits in the Euro Area, i.e. we hypothesize that historical movements of macro dynamics are influenced by waves of optimism and pessimism. From this, we have analyzed the effects of group behavior on the output and inflation gap. Indeed, our results support the behavioral framework by De Grauwe (2011), who assumes divergence in beliefs about the future value of both variables. The corresponding decision rules for market optimists and pessimists are described by the forecast performance of heterogeneous agents based on discrete choice theory. Furthermore, we contrast it with a standard hybrid version of the three-equations NKM under consideration of rational expectations. To the best of our knowledge, such kind of empirical study – a structural estimation of a bounded rationality model under consideration of the moment conditions – has not been extensively investigated before in the literature.

One of the main findings in this paper shows that a bounded rationality model with cognitive limitation provides a reasonable fit to auto- and cross-covariances of the data. Therefore our empirical results for the BR model offer new insights into expectation formation processes for the Euro Area. First, the agents had expected moderate deviations of the output gap from its steady state value over the whole time interval. Second, in the absence of rational behavior we find strong evidence for a backward-looking expectation formation process regarding the inflation gap. These results indicate that the market behavior acts as an amplification effect for a high degree of persistence in the data; i.e. animal spirits may strengthen the optimists' belief about the future output gap to diverge in the historical Euro Area data.

However, the estimation of the BR model (i.e. under the assumption of the divergence in beliefs) suggests that the parameters which measures the degree of habit formation in the dynamic IS curve, the standard deviation in the nominal interest rate shock and, unfortunately, the degree of the divergence in the movement of economic activity are *statistically insignificant*. In the latter case, however, no statement can be made about the uncertainty in agents' forecast regarding the output gap. A reason for this bounded rationality parameter being insignificant could be explained by the *highly non-linear* approach for modeling expectations being considered.

Nonetheless, this research will serve as a base for future studies. For example, one can further continue the model estimation with much richer models like e.g. the medium-scale version developed by Smets and Wouters (2005, 2007). More interestingly, different kinds of expectation processes can be considered for estimation. We leave all these issues to future research.

6 Appendix

6.1 A: Solution of the Baseline NKM under RE and BR

In general, all model specifications are described by the following system in canonical form:

$$AX_t + BX_{t-1} + CX_{t+1} + \varepsilon_t = 0, \tag{30}$$

where

$$X_t = \begin{pmatrix} y_t \\ \hat{\pi}_t \\ \hat{r}_t \end{pmatrix}, \; X_{t-1} = \begin{pmatrix} y_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \end{pmatrix}, \; X_{t+1} = \begin{pmatrix} \tilde{E}_t^j y_{t+1} \\ \tilde{E}_t^j \hat{\pi}_{t+1} \\ \tilde{E}_t^j \hat{r}_{t+1} \end{pmatrix}, \; \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{\hat{\pi},t} \\ \varepsilon_{\hat{r},t} \end{pmatrix}.$$

The corresponding matrices are given by:

$$A = \begin{pmatrix} 1 & 0 & \tau \\ -\lambda & 1 & 0 \\ -(1-\phi_{\hat{r}})\phi_y & -(1-\phi_{\hat{r}})\phi_\pi & 1 \end{pmatrix}, \; B = \begin{pmatrix} -\frac{\chi}{1+\chi} & 0 & 0 \\ 0 & -\frac{\alpha}{1+\alpha\nu} & 0 \\ 0 & 0 & -\phi_{\hat{r}} \end{pmatrix} \tag{31}$$

and

$$C = \begin{pmatrix} -\frac{1}{1+\chi} & -\tau & 0 \\ 0 & -\frac{\nu}{1+\alpha\nu} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{32}$$

Remember that for the *BR model* we assume

$$\begin{aligned} \tilde{E}_t^{BR} y_{t+1} &= (\alpha_{y,t}^O - \alpha_{y,t}^P) d_t \\ \tilde{E}_t^{BR} \hat{\pi}_{t+1} &= \alpha_{\hat{\pi},t}^{tar} \bar{\bar{\pi}} + \alpha_{\hat{\pi},t}^{ext} \hat{\pi}_{t-1} \end{aligned}$$

with

$$d_t = \frac{1}{2}(\beta + \delta\lambda_{y,t}), \tag{33}$$

where we consider the equations (10) to (18) with $\bar{\bar{\pi}} = 0$. In the following, we solve for the dynamics of the system (30). In case of the BR model, the 'solution' is given by

$$X_t = -A^{-1}[BX_{t-1} + CX_{t+1} + \varepsilon_t], \tag{34}$$

where the matrix A is of full rank, i.e. its determinant is not equal to zero, given the point estimates of the parameters in Table 1. Under consideration of the heuristics for the forecasts regarding the output and inflation gap expectations, the forward-looking term X_{t+1} is substituted by the equivalent expressions for the discrete choice mechanism given in section 2. It follows that the model is based on purely backward-looking behavior and thus (34) can be solved by backward induction. In particular, the backward solution of the BR model is based on a non-linear parameterization. To see this, the expected output gap can be rewritten as (cf. the equations (6) to (9)):

$$\tilde{E}_t^{BR} y_{t+1} = (2\alpha_{y,t}^O - 1)\frac{1}{2}(\beta + \delta\lambda_{y,t}). \tag{35}$$

In addition, the probability for being optimists can be rewritten as (cf. the equations (11) and (12)):

$$\begin{aligned} \alpha_{y,t}^O &= \frac{exp(\gamma U_t^O)}{exp(\gamma U_t^O) + exp(\gamma U_t^P)} = \frac{1}{1 + exp\{\gamma U_t^P - \gamma U_t^O\}} \\ &= \frac{1}{1 + exp\{\gamma (E_{t-1}^O y_t - y_t)^2 - \gamma (E_{t-1}^P y_t - y_t)^2\}}, \end{aligned} \tag{36}$$

where $U_t^P = -(E_{t-1}^P y_t - y_t)^2$ and $U_t^O = -(E_{t-1}^O y_t - y_t)^2$ are applied. Note here that no memory is assumed ($\rho = 0$).[22] Then we substitute the term $E_{t-1}^O y_t$ by $\frac{1}{2}(\beta + \delta\lambda_{y,t-1})$. Similarly, the term $E_{t-1}^P y_t$ is replaced by $-\frac{1}{2}(\beta + \delta\lambda_{y,t-1})$. After some algebra we get

$$\tilde{E}_t^{BR} y_{t+1} = \Big[2\Big\{\frac{1}{1 + exp\{-2\gamma(\beta + \delta\lambda_{y_{t-1}})y_t\}}\Big\} - 1\Big]\frac{1}{2}(\beta + \delta\lambda_{y,t}), \tag{37}$$

where the term inside the $[\cdot]$ brackets controls the magnitude of the changes in the divergence in beliefs, that is, $[\cdot] \in (-1, 1)$.

From this, we see that the analytic solution for y_t cannot be easily obtained due to the non-linearity in the expected future output gap of optimists. The same results hold for the derivation of the expected future output gap of pessimists. In addition, we obtain a non-linear solution formula for the expected inflation gap (not shown here), but the non-linearity can also not be dropped in this case. Hence, simulations are used to approximate the backward solution of the BR model. We estimate the BR model parameters by using SMM. In contrast, for the *RE model* we assume

$$\begin{aligned} \tilde{E}_t^{RE} y_{t+1} &= E_t y_{t+1} \\ \tilde{E}_t^{RE} \hat{\pi}_{t+1} &= E_t \hat{\pi}_{t+1}, \end{aligned}$$

where E_t denotes the mathematical expectation operator. As a result, the RE model is both backward- and forward-looking. Therefore we apply the method of undetermined coefficients in order to solve the model. The law of motion, which describes the analytic solution of the model, is given by

$$X_t = \Omega X_{t-1} + \Phi \varepsilon_t, \tag{38}$$

where $\Omega \in \mathbb{R}^{3\times 3}$ and $\Phi \in \mathbb{R}^{3\times 3}$ are the solution matrices. The former is a stable matrix as long as (similar to the matrix A in the BR case) its determinant is not equal to zero, which ensures the invertibility of Ω. Again, this is confirmed given

[22] It is based on our empirical results (cf. section 4.2). Note that equation (36) will be complicated in the case $\rho > 0$, while all non-linearity prevails.

the estimation results in Table 1. We substitute equation (38) into equation (30):

$$A(\Omega X_{t-1} + \Phi\varepsilon_t) + BX_{t-1} + C(\Omega X_t + \Phi E_t\varepsilon_{t+1}) + \varepsilon_t = 0.$$

This is equivalent to

$$A(\Omega X_{t-1} + \Phi\varepsilon_t) + BX_{t-1} + C(\Omega^2 X_{t-1} + \Omega\Phi\varepsilon_t + \Phi E_t\varepsilon_{t+1}) + \varepsilon_t = 0.$$

Hence, the reduced-form can be rewritten as

$$(C\Omega^2 + A\Omega + B)X_{t-1} + (A\Phi + C\Omega\Phi + I)\varepsilon_t = 0 \tag{39}$$

with I being the identity matrix. Note that $\varepsilon_t \sim N(0, \sigma_z^2)$ with $z = \{y, \hat{\pi}, \hat{r}\}$ and thus $E_t(\varepsilon_{t+1}) = 0$ holds. In order to solve equation (39), all the terms in brackets must be zero.[23] Thus the solution matrices can be uniquely determined. We may write that as

$$C\Omega^2 + A\Omega + B = 0 \Rightarrow \Omega = -(C\Omega + A)^{-1}B. \tag{40}$$

In order to solve the quadratic matrix equation (40) numerically, we apply the brute force iteration procedure (Binder and Pesaran (1995)). Hence an equivalent recursive relation of (40) is given by

$$\Omega_n = -(C\Omega_{n-1} + A)^{-1}B \tag{41}$$

with an arbitrary number of iteration steps N, i.e. $n = \{1, 2, ..., N\}$. We define as the initial value $\Omega_0 = \varsigma I$ with I being the identity matrix and $0 \le \varsigma \le 1$, where we set $\varsigma = 0.8$. The iteration process (41) proceeds until $||\Omega_n - \Omega_{n-1}|| < \varrho$ holds, where ϱ is an arbitrarily small number (we set $\varrho = 0.1^6$).[24] Given the 'solution' of Ω, the computation of Φ is straightforward:

$$A\Phi + C\Omega_n\Phi + I = 0 \Rightarrow \Phi = -(A + C\Omega_n)^{-1}. \tag{42}$$

6.2 B: Delta Method and Confidence Interval for Auto-/Cross-Covariances

The Delta method is a common technique for providing the first-order approximations to the variation of moments (see Chapter 5 of Davidson and MacKinnon (2004) among others). In this paper, we compute the standard errors of the estimated auto- and cross-covariances of the data via the Delta method. The covariance is defined as:

$$\gamma_{ij}(h) = E[(X_{i,t} - \mu_i)(X_{j,t+h} - \mu_j)'], \quad t = 1, \cdots, T, \tag{43}$$

where γ_{ij} is the auto-covariance function when $i = j$. Otherwise γ_{ij} denotes the cross-covariance between $X_{i,t}$ and $X_{j,t+h}$. h and μ_i(or μ_j) are the lag length

[23] It is obvious that we can discard the trivial solution $X_{t-1} = \Gamma_t = \varepsilon_t = 0$.

[24] In particular, we search for the fix point of Ω_n according to equation (41), respectively, such that $\Omega_N = f(\Omega_N) = -(C\Omega_N + A)^{-1}B$ holds.

and the sample mean of the variable X_i(or X_j), respectively. The covariance function in Equation (43) proceeds with a simple multiplication:

$$\gamma_{ij}(h) = E[X_{i,t} \cdot X'_{j,t+h}] - \mu_i \cdot E[X'_{j,t+h}] = \mu_{ij} - \mu_i \cdot \mu_j,$$

where μ_{ij} denotes $E[X_{i,t} \cdot X'_{j,t+h}]$. Now, we see that $\gamma_{ij}(h)$ is a transformed function of the population moments μ_i, μ_j and μ_{ij}. The vector μ is denoted as the collection of the moments: $\mu = [\mu_i \quad \mu_j \quad \mu_{ij}]$. The covariance function can be differentiated with respect to the vector μ, as follows:

$$D = \frac{\partial \gamma_{ij}(h)}{\partial \mu} = \begin{bmatrix} \frac{\partial \gamma_{ij}(h)}{\partial \mu_i} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_j} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_{ij}} \end{bmatrix} = \begin{bmatrix} -\mu_j \\ -\mu_i \\ 1 \end{bmatrix}. \tag{44}$$

Note that the Delta method is used to provide the asymptotic distribution for the estimate $\widehat{\gamma}_{ij}$ when matching the sample moments of the data:

$$\sqrt{T}(\gamma_{ij} - \widehat{\gamma}_{ij}) \ \sim \ N(0, D'SD), \tag{45}$$

where $D'SD$ is the covariance matrix of the estimated moments. As regards to some suitable lag length q, we employ a common HAC estimator when estimating the covariance matrix of sample moments:

$$\widehat{\Sigma}_\mu = \widehat{C}(0) + \sum_{k=1}^{q} \Big(1 - \frac{k}{q+1}\Big)[\widehat{C}(k) + \widehat{C}(k)'] \tag{46}$$

with

$$\widehat{C}(k) = \frac{1}{T} \sum_{t=k+1}^{T} [f(z_t) - \widehat{\mu}][f(z_{t-h}) - \widehat{\mu}]', \tag{47}$$

where $f(z_t) = [X_i,\ X_j,\ X_i \cdot X_j]$ holds. The total number of lags is once again denoted by k. In particular, we follow the advice by Davidson and MacKinnon (2004, p. 364) and scale q with $T^{1/3}$. Accordingly, we set it to $q = 5$ for the Euro Area data. The optimal weight matrix is defined as $S = \widehat{\Sigma}_\mu^{-1}$, which will be used to estimate the covariance matrix of moments. If we use s_γ to denote $\sqrt{D'SD}$, then the 95% asymptotic confidence intervals for auto- and cross-covariance estimates can be expressed as:

$$[\gamma_{ij} - 1.96 \cdot s_\gamma, \quad \gamma_{ij} + 1.96 \cdot s_\gamma]. \tag{48}$$

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